1) A binary channel with equally likely transmitted symbols, with transmitter X and receiver Y, has the following conditional probabilities:

\[ P(Y = 0 \mid X = 0) = 0.9 \]
\[ P(Y = 1 \mid X = 0) = 0.1 \]
\[ P(Y = 0 \mid X = 1) = 0.4 \]
\[ P(Y = 1 \mid X = 1) = 0.6 \]

1a) **Compute** \( P(Y = 0) \) [Answer: 0.65]
1b) **Compute** \( P(Y = 1) \) [0.35]
1c) [15] Being equally likely, the entropy of the input \( H(x) = 1 \) bit/symbol. **Compute** \( H_{\text{eff}}(Y) \), the effective information transfer rate seen by the output. NOTE & HINT: Equation 9.10 in the text has the Equivocation with a minus sign in the front. *That is correct.* I inadvertently left off the minus sign when covering this in class. [0.2142 bits/symbol]

2) [25] Which of the following two systems would have the lower \( P(\text{Bit Error}) \)? (Show your computations). Both systems use Gray Coding whereby adjacent symbols differ by only 1 bit, and both systems use Matched Filter Detectors (MFD).

**System 1:** Coherently detected 4-ary PSK (a.k.a. QPSK) with \( \frac{E_b}{N_0} = 13 \) dB.

**System 2:** Coherently detected 8-ary PSK with \( \frac{E_b}{N_0} = 16 \) dB.

[QPSK has \( P(\text{BE}) = 133.3(10^{-12}) \). 8-PSK has a symbol error rate of \( 3.338(10^{-9}) \) and bit error rate of \( \approx 1.113(10^{-9}) \). Hence QPSK is better.]

3) A BPSK signal \( x(t) = m(t)\cos(2\pi 4,400,000t) \), where \( m(t) \) is a zero mean, 7.6 watt, 10 Kbps random binary square wave.

3a) [15] **Find** the time average autocorrelation \( R_{XX}(\tau) \). \([0.5 R_{MM}(\tau)\cos(2\pi 4,400,000\tau)]\)

3b) [10] **Sketch** the power spectrum \( G_X(f) \). [You should sketch \( \text{sinc}^2 \) functions centered on \( \pm 4.4 \text{ MHz} \). The height of the functions is \( 190(10^{-6}) \), and nulls are at integer multiples of 10 KHz from the centers.]

4) A satellite transmitting a BPSK signal at a center frequency of 3 GHz, with 26 watts power out, is located 38,000 km from an earth receiver. The coherent MFD receiver is known to have a system temperature of 700 degrees K. Other losses are known to be 1.8 dB. Both the satellite and receiver have 4 meter diameter parabolic antennas, with an antenna efficiency of 0.59.
[25] **Compute** the maximum bit rate the system could support if a $P(\text{BE}) \leq 16.95(10^{-9})$ is desired. Note $Q(5.200) = 16.95(10^{-9})$. [504.7 Mbps]