1) A 1,000 bps binary system uses Raised Cosine pulses for Logic 0's and Logic 1's. The symbol for a logic 0 during a bit interval is $s_0(t) = 1 - \cos(2\pi 1000t); 0 \leq t \leq T$, $T$ being the bit time $= 1/1000$ second. The symbol for a logic 1 during a bit interval is $s_1(t) = -s_0(t) = \cos(2\pi 1000t) - 1; 0 \leq t \leq T$. Assume a random sequence of these symbols, contaminated by 200 mw of Additive White Gaussian Noise, is input into a Single Sample Detector, which samples each input bit once at time $t = T/2$.

1a) [2] **Compute** the average voltage out of the sampler at time $T/2$, assuming a Logic 0 was transmitted.

1b) [2] **Compute** the average voltage out of the sampler at time $T/2$, assuming a Logic 1 was transmitted.

1c) [3] **Compute** the noise power out of the sampler at time $T/2$.

1d) [3] **Compute** the P(Bit Error) for this system.

\[ P(\text{BE}) = 2 \times \frac{1}{2} \left( \frac{2}{\sqrt{2}} \right) = \left( 4.472 \right) = 3.875 \times 10^{-6} \]
2) A 50,000 bps binary communications system transmits a symbol \( s_0(t) = 50,000t; 0 \leq t \leq T \) for a logic 0, and \( s_1(t) = -50,000t; 0 \leq t \leq T \) for a logic 1. The signals are contaminated by zero mean, 1.20 watt, thermal noise.

2a) [3] Assume the thermal noise is modeled by Band-Limited White Gaussian Noise with a maximum frequency of 1.000 GHz. Compute the two sided noise power spectral density \( N_0/2 \).

2b) [7] If a Matched Filter Detector is used at the receiver, compute the P(Bit Error). Model the noise as White Gaussian Noise and use the \( N_0/2 \) value calculated in 2a.

\[
\begin{align*}
1.2w &= \frac{N_0}{2} \left( \frac{1}{1,000} \right) (10^9) \\
N_0 &= \frac{600(10^{-15})}{2} = \text{ANS}
\end{align*}
\]
\[
\text{Noise} = E \left[ S_0^T (t) [S_0(t) - S_1(t)] \right] dt \cdot S_0^T (t) [S_0(t) - S_1(t)] dt
\]

\[
= \int S_0^T \int R_{NN} (r-t) [S_0 (t) - S_1 (t)] [S_0 (r) - S_1 (r)] \, dt \, dr
\]

where \( R_{NN} (r-t) = \frac{N_0}{2} s(r-t) \)

\[
= \frac{N_0}{2} \int S_0^T (S_0 (t) - S_1 (t))^2 \, dt = \frac{N_0}{2} \int (\alpha^2 t^2 + 2\alpha^2 t^2 + \alpha^2 t^2) \, dt
\]

\[
= 2N_0 \alpha^2 \int t^2 \, dt = 2N_0 \alpha^2 T^3 \left. \frac{T}{3} \right|_0^T = 2N_0 \alpha^2 T^3 \frac{T}{3} = \frac{2}{3} N_0 T \text{ watts}
\]

\[
\text{P(BE)} = 2 \cdot \frac{1}{2} \cdot \Phi \left( \frac{\sqrt{\frac{4}{3}T^2}}{\frac{N_0}{\sqrt{3}}} \right) = \Phi \left( \sqrt{\frac{4T^2}{3N_0}} \right)
\]

\[
= \Phi \left( \frac{\sqrt{12}}{\sqrt{18N_0}} \right) = \Phi \left( \frac{\sqrt{12}}{\sqrt{18}} \cdot \frac{1}{50,000} \cdot \frac{1}{1200} (10^{-15}) \right)
\]

\[
= \Phi \left( \frac{1.1011 \times 10^6}{1.1011 \times 10^6} \right) = \Phi \left( 3333 \right) \approx 0.685
\]