The purpose of the project is to help you familiarize with computer aided control system analysis using MATLAB. Please show all your steps clearly and include all MATLAB programs used to show your work.

Consider the automobile suspension system shown in the figure below. Mass $M_1$ is equal to the effective mass of the wheel and the mass $M_2$ is equal to one fourth of the mass of the automobile frame. The spring constant $K_1$ is the stiffness of the automobile suspension spring, $K_2$ is the stillness of the tire, and viscous coefficient $b_1$ is the damping constant of the shock absorber. The input $x(t)$ is the ground level and the output $y(t)$ is the vertical position of the automobile frame relative to some equilibrium position. The vertical position of the wheel relative to the equilibrium position is $q(t)$. It is assumed that the bottom of the tire remains in contact with the road surface at all the time. The gravitation forces acting on the frame and the wheel are neglected.

A. Modeling

(1) Derive a mathematical model for the system.
(2) Get the transfer function between $Y(s)$ and $X(s)$
B. MATLAB programming

(3) Using the transfer function from (2), create a transfer function object and find the poles for the system by using the following parameters and the `roots` command in MATLAB:

\[ M_1 = 10 \text{kg} , \ M_2 = 50 \text{kg} , \ k_1 = 100 \text{N/m} , \ k_2 = 30 \text{N/m} , \ b_1 = 120 \text{N} \cdot \text{s/m} \]

(4) Based on the poles, guess the impulse response of the system. Explain the reason of your guess. (Do not use MATLAB for this problem).

(5) Draw all the zeros and poles by using `pzmap` command in MATLAB.

(6) Get the impulse, step and sinusoid responses for the system. (Use `impulse`, `step`, and `gensig` MATLAB commands to generate the inputs).

(7) Get the response of the system for the following input:

\[ x(t) = \begin{cases} 
0 & 0 \leq t < 1 \\
0.05 \times (t-1) & 1 \leq t < 3 \\
0.15 & 3 \leq t < 10 \\
0 & 10 \leq t 
\end{cases} \]

(8) Get discrete transfer function at the following sampling periods:
\[ T=0.01, \ T=0.25 \text{ and } T=1. \]

(9) Plot the poles and zeros of the discrete transfer functions from (8).

(10) Get impulse responses of the discrete transfer functions resulted from (8). Plot them. Compare the impulse responses resulted from continuous transfer function and discrete transfer functions.