Consider a DC motor and a dynamometer connected by a flexible coupling as shown in the figure below. Dynamometer is used to generate torque to test DC motor performance. The schematic of the system is shown in Fig. 1. As can be seen, this system consists of electrical and mechanical components.

Fig. 1 Schematic diagram of DC motor with Load

1. Get a mathematical model for the motor (In this system, $T=K\cdot i$ where $K$ is armature constant, $i$ is armature current. $e$ is back emf, $k$ is the spring constant representing the flexible coupling, $J$ is the moment of inertia of motor, $J_2$ is the moment of inertia of dynamometer)

2. Get transfer function between input voltage $V$ and angular displacement $\theta_2$
3. Rewrite the transfer function from problem 2 into differential equation and get impulse response analytically using the values listed below (you may simplify the equation by ignoring small numbers after plugging in values):

- moment of inertia of the motor $J_1=3.2284\times10^{-6}$ kg$m^2$/s$^2$
- moment of inertia of the dynamometer $J_2=1\times10^{-6}$ kg$m^2$/s$^2$
- spring constant $k_1=1\times10^{-2}$ N$m$/rad
- damping constant $b=3.5077\times10^{-3}$ N$s$/m$^2$/rad
- electromotive force constant ($K=K_e=K_t$) $=0.0274$ N$m$/Amp
- electric resistance ($R$) $= 4$ ohm
- electric inductance ($L$) $= 2.75\times10^{-6}$ H

4. Use MATLAB to get impulse response of the system. Compare the result with the analytical solution in problem 3.

5. Let's consider a mass-spring-damper system. First, get a mathematical model of this system. Then use MATLAB to get impulse response and pole. Try this with at least 10 different values. Discuss the relationship between pole locations and impulse response of the system. (Use pzmap command to draw all the pole locations in one plot)

6. Consider the transfer function in problem 2. Get the transfer function between input voltage $V$ and output angular velocity $\omega$. Vary some of the parameters of the motor so that you will get different poles. Use MATLAB to get step response. Repeat this with at least 10 times. Explain the relationship between pole location and response. Draw a pole plot containing all poles that you considered during the experiment and make comments on the plot. Compare the result with problem 5.

7. Discretize the transfer function in problem 2 using the approximate discretization method taught in the class. Compare the discrete transfer function with the discretized transfer function by MATLAB in terms of poles and zeros with several different sampling periods.

8. Simulate the discretized system at several different sampling periods. Compare it with continuous time model. What is the slowest sampling rate that we can use for the given system?