Choose any four out of five problems. 

Please specify which four listed below to be graded:
1)_____; 2)_____; 3)_____; 4)_____; 

Name: ______________________________

Student ID: ________________________________

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Problem 1:
Consider the system
\[ x(k+1) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ 3 \end{bmatrix} u(k) , \]
\[ y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(k) \]
and let \( x(0) = 0 \) and \( u(k) = 1, n \geq 0 \).

a) Determine \( \{y(k)\}, k \geq 0 \) by any approach.

b) If it is known that when \( u(k) = 0 \), then \( y(0) = y(1) = 1 \), can \( x(0) \) be uniquely determined? If your answer is affirmative, determine \( x(0) \).
Problem 2:

Let

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & 1 & 0 \\
0 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & -1 & 0 \\
\end{bmatrix},
\]

Find $e^{At}$ and $\sin At$.
Problem 3:
Show that there exists a similarity transformation matrix $P$ such that

$$P A P^{-1} = A_c = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
-\alpha_0 & -\alpha_1 & -\alpha_2 & \cdots & -\alpha_{n-1}
\end{bmatrix},$$

if and only if there exists a vector $b \in \mathbb{R}^n$ such that the rank of $[b \ Ab \ \cdots \ A^{n-1}b]$ is $n$. 

**Problem 4:**
Consider the matrix

\[
A = \begin{bmatrix}
-\alpha_1 & -\alpha_2 & -\alpha_3 & -\alpha_4 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix},
\]

show that its characteristic polynomial is given by

\[
\Delta(\lambda) = \lambda^4 + \alpha_1\lambda^3 + \alpha_2\lambda^2 + \alpha_3\lambda + \alpha_4.
\]

Show also that if \( \lambda_i \) is an eigenvalue of \( A \), then \( [\lambda_i^3 \lambda_i^2 \lambda_i 1]^T \) is an eigenvector of \( A \) associated with \( \lambda_i \).
**Problem 5:**
Consider the system representations given by

\[
x(k + 1) = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(k)
\]

\[
y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 & 0 \end{bmatrix} u(k)
\]

and

\[
\tilde{x}(k + 1) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)
\]

\[
y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} 0 & 1 \end{bmatrix} u(k)
\]

Are these representations equivalent? Are they zero-input equivalent?