ECEN 4503
Random Signals and Noise
Spring 2002
Final Exam

Choose any four out of five problems,
*Please specify*
1)_____; 2)_____; 3)_____; 4)_____

Name: ______________________________

Student ID: ______________________________

E-Mail Address: ______________________________
**Problem 1:**
In a computer simulation it is desired to transform numbers, that are values of a random variable $X$ uniformly distributed on $(0,1)$, to numbers that are values of a Cauchy random variable $Y$ as defined by

$$F_Y(y) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1}\left(\frac{y}{b}\right).$$

Find the required transformation $T$. 
**Problem 2:**
The random variables $X$ and $Y$ are statistically independent with exponential densities

\[ f_X(x) = \alpha e^{-\alpha x} u(x), \text{ and} \]
\[ f_Y(y) = \beta e^{-\beta y} u(y). \]

Find the probability density function of the random variable $Z = \min(X, Y)$.
**Problem 3:**
The random variables $X$ and $Y$ are statistically independent with Rayleigh densities

\[ f_X(x) = \frac{x}{\alpha^2} e^{-x^2/2\alpha^2} u(x), \text{ and} \]
\[ f_Y(y) = \frac{y}{\beta^2} e^{-y^2/2\beta^2} u(y). \]

Show that if $Z = X / Y$, then

\[ f_Z(z) = \frac{2\alpha^2}{\beta^2} \frac{z}{(z^2 + \alpha^2 / \beta^2)^2} u(z). \]
Problem 4:
A random process is defined by
\[ Y(t) = X(t) \cos(\omega_0 t + \Theta), \]
where \( X(t) \) is a wide-sense stationary random process that amplitude-modulates a carrier of constant angular frequency \( \omega_0 \) with a random phase \( \Theta \) independent of \( X(t) \) and uniformly distributed on \((−\pi, \pi)\). Find \( E[Y(t)] \) and autocorrelation function of \( Y(t) \). Is \( Y(t) \) wide-sense stationary?
**Problem 5:**
A random process is given by
\[ X(t) = A \cos(\Omega t + \Theta) \]
where \( A \) is a real constant, \( \Omega \) is a random variable with density function \( f_{\Omega}(\cdot) \), and \( \Theta \) is a random variable uniformly distributed on the interval \((0, 2\pi)\) independently of \( \Omega \). Show that the power spectrum of \( X(t) \) is
\[ S_{xx}(\omega) = \frac{\pi A^2}{2} [f_{\Omega}(\omega) + f_{\Omega}(-\omega)]. \]