Classification of Systems (20%) 

Problem 1a) (10%) Consider the memoryless systems given below, in which \( u \) denotes the input and \( y \) is the output. Which of them is a linear system? If not, is it possible to introduce a new output such that the system becomes linear?

i) \[ y = u \]

ii) \[ y = u + y_0 \]

iii) \[ y = u^2 \]

Problem 1b) (10%) The impulse response of a linear system is found to be \( g(t, \tau) = e^{-|t-\tau|} \) for all \( t \) and \( \tau \). Is this system causal? Is it time-invariant? (Hint: \( y(t) = \int_{-\infty}^{\infty} g(t, \tau)u(\tau)d\tau \))

System Representation (20%) 

Problem 2 Find all three representations (i.e., input-output operator, transfer function, and state space equations) of the following RLC circuit,

Linearization (20%) 

Problem 3 A nonlinear system is given by

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
f_1(x_1, x_2, u) \\
f_2(x_1, x_2, u)
\end{bmatrix} = \begin{bmatrix}
1 + 2e^{2x_1} - 3(x_2 - 1)^2 + \sin(5u) \\
\frac{1}{3}x_1x_2^3 - x_1x_2 + 2\ln(1 + x_1)
\end{bmatrix}.
\]

Note that \( x = [0 \ 0]^T \) is an equilibrium point at \( u = 0 \). Linearize the system about the equilibrium point. To improve the accuracy, approximate up to the second order in the linearization process (instead of only the first order: \( \nabla_x f, \nabla_u f \) ) in Taylor series expansion. Find the linearized system (my be not in the form of \( \{A, B, C, D\} \)).
Realization (20%)

*Problem 4a* (10%) Find an irreducible (i.e., minimal) controllable canonical form realization (i.e., its simulation diagram and state space equations) for the following system,

$$H(s) = \begin{bmatrix} 2s \\ s^3 + 6s^2 + 11s + 6 \\ s^2 + 2s + 2 \\ s^4 + 6s^3 + 9s^2 + 4s \end{bmatrix}$$

(hint: A is $6 \times 6$).

*Problem 4b* (10%) Find the $\{A, B, C, D\}$ matrices of the composite interconnected system given below,

$$\begin{align*}
\dot{x} &= A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + B u_a; \\
y_a &= C \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + D u_a
\end{align*}$$

$s_H \equiv \{A_i, B_i, C_i, D_i\}, i = 1, 2, 3, 4$ (hint: you may stop at the temporary variables which are functions of $\{A_i, B_i, C_i, D_i\}, i = 1, 2, 3, 4$).

Linear Algebra (15%)

*Problem 5a* (5%) Is it possible to define rules of addition and multiplication such that the set $\{0, 1, 2\}$ forms a field?

*Problem 5b* (5%) The set of all $2 \times 2$ matrices with the usual definitions of matrix addition and multiplication is clearly *not* a field. Now consider the set of all $2 \times 2$ matrices of the form

$$\begin{bmatrix} x & -y \\ y & x \end{bmatrix}$$
where \( x \) and \( y \) are arbitrary real numbers (i.e., \( x, y \in \mathbb{R} \)). Does the set with the usual definitions of matrix addition and multiplication form a field? If not, show why? If yes, what are the zero and unity elements?

**Problem 5c** (5%) Consider the matrix

\[
A = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
-\alpha_n & -\alpha_{n-1} & -\alpha_{n-2} & \cdots & -\alpha_1
\end{bmatrix}
\]

find 1) \(|A|\), 2) \(\text{trace}(A)\), 3) \(r(A)\), and verify 4)

\[
A^{-1} = \begin{bmatrix}
-\alpha_{n-1}/\alpha_n & -\alpha_{n-2}/\alpha_n & \cdots & -1/\alpha_n \\
1/\alpha_n & 1/\alpha_n & \cdots & -1/\alpha_n \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0
\end{bmatrix}
\]

**Evaluation (5%)**

**Problem 6a** Please rank the difficulty of the problems in Midterm exam #1:
Easiest ○ ○ ○ ○ ○Moderate ○ ○ ○ ○○Hardiest

**Problem 6b** Were homework assignments require a reasonable amount of time and effort:
Too simple ○ ○ ○ ○ ○Just right ○ ○ ○ ○○Too demanding

**Problem 6c** Were homework assignments:
Too easy ○ ○ ○ ○ ○Just right ○ ○ ○ ○○Too difficult

**Problem 6d** Describe the pace of the progress:
Too slow ○ ○ ○ ○ ○OK ○ ○ ○ ○○Too fast

**Problem 6e** Describe the course content:
Boring ○ ○ ○ ○ ○OK ○ ○ ○ ○○Interesting

**Problem 6f** Any other suggestions? hope? or wish?