ECEN 5713 Linear Systems
Spring 2001
Midterm Exam #1

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Problem 1:
Suppose we have a state-space realization given by $A, b, c$ with the three chosen state variables
$$x = [x_1, x_2, x_3]^T.$$ Suppose we are now interested in the state variables $z = [z_1, z_2, z_3]^T$, where
$$z_1 = k_1 x_1, z_2 = k_2 x_2, \text{ and } z_3 = k_3 x_3,$$ and we let $\dot{z} = Fz + gu, \quad y = hz$.

a) Write out the matrices $F, g, h$ in terms of the elements of $A, b, c$ and the scale factors $k_1, k_2, k_3$.

b) Suppose we wish to change the time scale and substitute $\tau = a_0 t$ into the equations. Repeat part a), showing how $F, g, h$ depend on the time scale factor $a_0$ and the elements of $A, b, c$. 
Problem 2:  
If $\{A, b, c, d\}, d \neq 0$, is a realization with $H(s) = c(sI - A)^{-1}b + d$, show that $\{A - (bc/d), b/d, -c/d, 1/d\}$ is a realization for a system with transfer function $1/H(s)$. 
Problem 3:
Realize the following SIMO continuous-time, time-varying system and show one feasible state space representation, i.e., \( \{ A(t), B(t), C(t), D(t) \} \),
\[
\begin{align*}
    e^{-\tau} \dot{y}_1(t) + y_1(t) + \dot{y}_2(t) + y_2(t) &= tu(t) \\
    \dot{y}_1(t) + \dot{y}_2(t) + ty_2(t) &= \ddot{u}(t) + t^2u(t)
\end{align*}
\]
Problem 4:

A nonlinear system is given by
\[
\dot{x} = \begin{bmatrix}
    \dot{x}_1 \\
    \dot{x}_2 \\
\end{bmatrix} = \begin{bmatrix}
    f_1(x_1, x_2, u) \\
    f_2(x_1, x_2, u) \\
\end{bmatrix} = \begin{bmatrix}
    1 + 2e^{x_1 - 3(x_2 - 1)^2} + \sin 5u \\
    \frac{1}{3} x_1 x_2^3 - x_1 x_2 + 2 \ln(1 + x_1) \\
\end{bmatrix}.
\]

Linearize the system about the equilibrium point. To improve the accuracy, approximate up to the second order in the linearization process.
Problem 5:
Let
\[
H(s) = \begin{bmatrix}
\frac{s^2 + 1}{s^3} & \frac{2s + 1}{s^2} \\
\frac{s + 3}{s^2} & \frac{2}{s}
\end{bmatrix}
\]
be a transfer function matrix. Find a minimal realization (i.e., simulation diagram and state space representation) for the continuous-time system defined above as, \( H(s) \).