ECEN5533 Quiz #2 25 October 2011 Name Solution

Async Distance Learning take by 1 November

$\omega_1 \triangleq 2\pi 1000$
$\omega_2 \triangleq 2\pi 2000$

1) A signal $x(t) = 10\cos(2\pi 1000t) + 20\cos(2\pi 2000t)$ is to be sampled and quantized.

1a) [2] Assuming ideal sampling, what is the minimum allowable time interval between sample values that will allow perfect signal reproduction?

1b) [7] The sampled signal is then fed to a uniform quantizer with L levels that must not saturate, L some power of 2, converted to a binary bit stream via Pulse Code Modulation (PCM), and possibly transmitted to an information sink. What would be the bit rate if the quantizer SNR was desired to be at or greater than 40 dB, but as close to 40 dB as possible?

1c) [2] Suppose you have 1900 Hz of bandwidth available to move the sampled and quantized signal to this information sink. Theoretically, what's the maximum possible symbol rate that can be moved over this bandwidth?

1d) [2] If M-ary signaling is used, what is the minimum value of M (M some power of 2) that could be used to transmit the quantizer output?

1e) [7] Regardless of the answer you got in 1d, suppose a zero mean 4-ary Pulse Amplitude Modulated signal is to be used, and it's known that this signal will be contaminated with 1 milliwatt of Additive White Gaussian Noise (AWGN) at the receiver. What is the minimum amount of signal power that must be at the receiver in order to insure the P(*symbol* error) is less than 0.0042? Assume a *single sample detector* is to be used, and the M pulses have equally spaced amplitudes.

(1a) Max frequency is 2000 Hz. A sampling frequency $\geq$ to & > 4000 Hz is required. $\Rightarrow T_s$ a little less than $\frac{1}{4000}$ needed. ANS

(1b) $P_{SIG} = \lim_{T\to\infty} \frac{1}{T}\int_T x^2(t)dt = \lim_{T\to\infty}\frac{1}{T}\int_T (100\cos^2\omega_1 t + 200\cos\omega_1 t\cdot\cos\omega_2 t + 400\cos^2\omega_2 t)dt$

$= \lim_{T\to\infty}\frac{1}{T}\int_T (50+200)dt = 250$ watts

$P_{NOISE} = \frac{\Delta V^2}{12}$ Want $\frac{250\cdot 12}{\Delta V^2} \geq 10{,}000 \Rightarrow .5477 > \Delta V$

MAX input swing is ± 30v $\Rightarrow \frac{60}{\Delta V} = 109.5$

$\Rightarrow$ need L= 128 levels

$\Rightarrow$ 7 bits per level

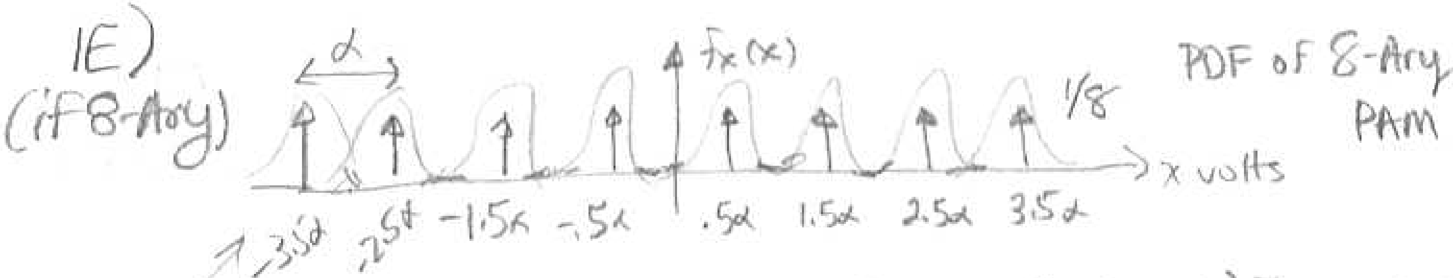
$\Rightarrow$ binary bit rate $= 4000 \frac{\text{samples}}{\text{sec}} \times 7 \frac{\text{bits}}{\text{sample}} = 28$ Kbps ANS

1C) Nyquist showed that, with ideal filtering, 1900Hz of BW can handle 3800 symbols per second.
ANS

1D) Need to move $\frac{28{,}000 \text{ bits/sec}}{3{,}800 \text{ symbols/sec}} = 7.368 \frac{\text{bits}}{\text{symbol}}$.

$M = 256$ (8 bits/symbol) will work
ANS

1E) (if 8-Ary)



$$\text{Power} = \frac{1}{4}\left[(.5\alpha)^2 + (1.5\alpha)^2 + (2.5\alpha)^2 + (3.5\alpha)^2\right] = E[X^2]$$

$$= \frac{\alpha^2}{4}(.25 + 2.25 + 6.25 + 12.25) = 5.25\,\alpha^2 \text{ watts}$$

Want P(symbol error) < .0042

$= 14\,P(\text{symbol A is in error}) \Rightarrow P(\text{symbol A in error}) < .0003$

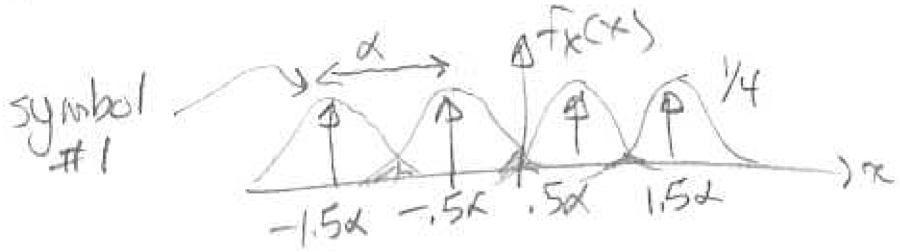
$\Rightarrow Q(3.44) < .0003$, or argument > 3.44

$$Q\left(\frac{\alpha/2}{\sqrt{.001}}\right) = Q\left(\frac{\alpha}{.06325}\right) \Rightarrow \text{want } \frac{\alpha}{.06325} > 3.44$$

$\alpha > .2176$

$\Rightarrow \text{Power} \geq 5.25(.2176^2) = .2485$ watts
ANS

1E) per problem statement



$$\text{Power} = E[X^2] = \frac{1}{2}\left[(.5\alpha)^2 + (1.5\alpha)^2\right] = \frac{\alpha^2}{2}[.25 + 2.25]$$

$$= 1.25\alpha^2 \text{ watts}$$

Want P(symbol error) $\leq .0042$

6 P(symbol #1 is in error) $\leq .0042$

P(symbol #1 is in error) $\leq .0007$

$$Q\left(\frac{\alpha/2}{\sqrt{.001}}\right) = Q\left(\frac{\alpha}{.06325}\right) \leq .0007$$

or $$\frac{\alpha}{.06325} \geq 3.20$$

$$\alpha \geq .2024$$

$\Rightarrow$ Power = .0512 watts ANS

Note 8-Ary requires ≥ 248.5 SNR, while 4-Ary requires 51.2 SNR. Binary is even less. 256-Ary needs much higher SNR.