ECEN5533 Quiz #1 **Name** Solution

Live: 13 September 2011 **Distance Learning: no later than 20 September 2011**

1) Given the power spectrum $G_X(f) = 1 - |f|/32{,}000{,}000$; $|f| \le 32{,}000{,}000$ Hz, find the values of the

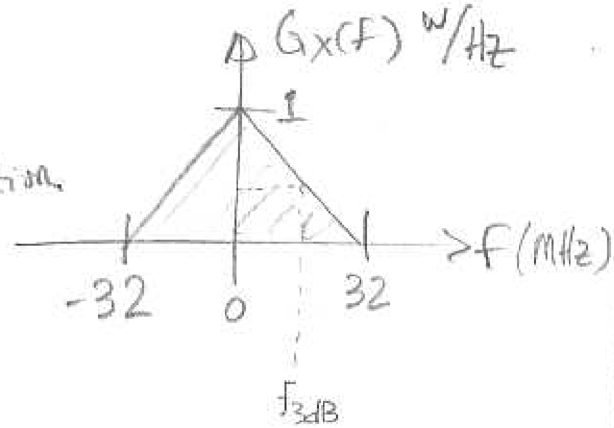
1a) [2] Half-power bandwidth

1b) [2] Noise Equivalent bandwidth

1c) [2] 99% of power bandwidth

1d) [2] Bandwidth beyond which the attenuation is $\ge$ 35 dB

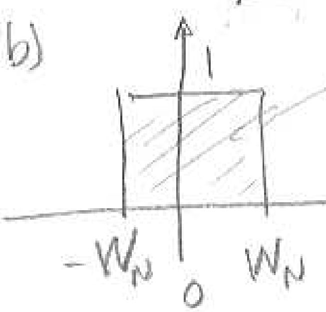
1e) [2] Absolute bandwidth $\Rightarrow$ $\underline{32\text{MHz}}$ ANS By Inspection.



Area = $32(10^6)$

1a) $\underline{16\text{MHz}}$ ANS by inspection

1b)



want this area = area under triangle

$\Rightarrow W_N = \underline{16\text{MHz}}$ ANS

NOTE! ½ power BW & Noise BW are frequently $\underline{\text{not}}$ equal.

1c) Want $2\int_0^{\alpha}\left(1 - \frac{f}{32(10^6)}\right)df = 99\%$ of area $= 31.68(10^6)$

$$\Rightarrow f - \frac{f^2}{64(10^6)}\Big|_0^{\alpha} = 15.84(10^6) \Rightarrow f^2 - 64(10^6)f + 1.014(10^{15}) = 0$$

$$f = \frac{64(10^6) \pm \sqrt{4.096(10^{15}) - 4.055(10^{15})}}{2}$$

$= 32(10^6) \pm 3.202(10^6)$

$\Rightarrow f = \underline{28.8(10^6)\,\text{Hz}}$ ANS

1D) $35 = 10\log\alpha$

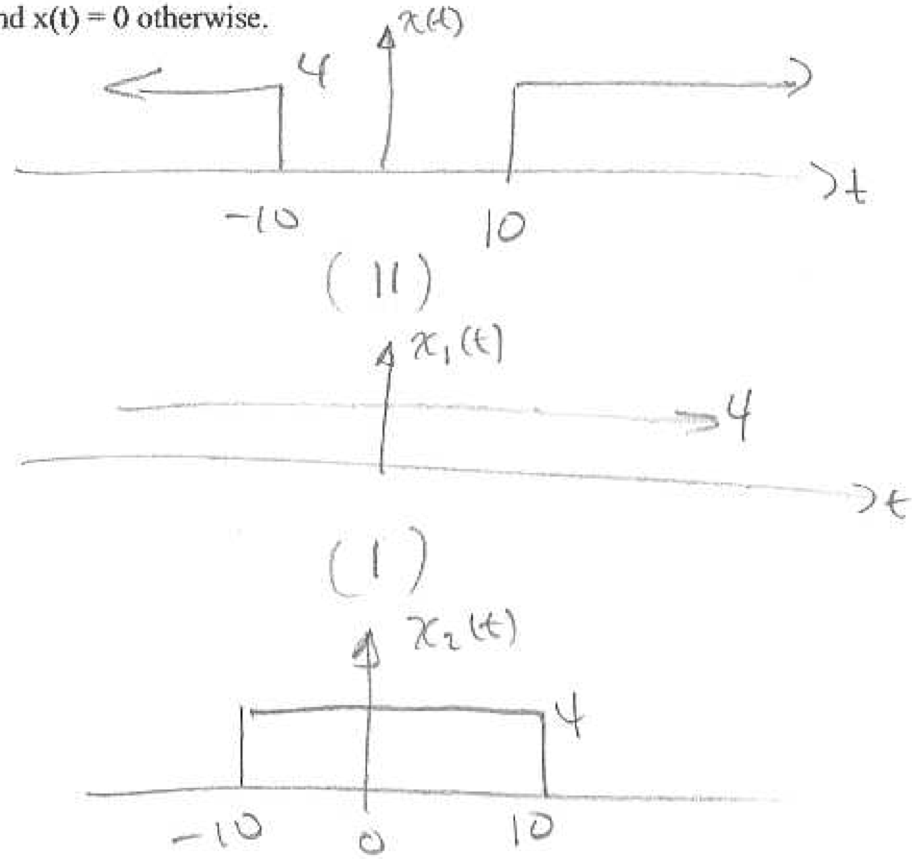
$\alpha = 3{,}162$

$\Rightarrow G_X(f) \overset{\text{WANT}}{\le} \frac{1}{3162}$

$1 - \frac{f}{32(10^6)} \le \frac{1}{3162}$

$\Rightarrow f \ge \underline{31.99\text{ MHz}}$ ANS

2) Suppose x(t) = 4 for | t | > 10, and x(t) = 0 otherwise.
[10] Find X(f).



$X(f) = X_1(f) - X_2(f)$

$= 4\delta(f) - 80\,\mathrm{sinc}\,20f$

ANS

or

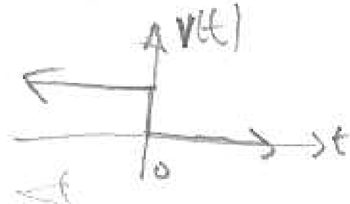
$x(t) = u(t-10) + u(-t-10)$

step occurs where argument $= 0$

$u(t) \Leftrightarrow \frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$ (#9 Table A.6) $= U(f)$

$u(t-10) \Leftrightarrow \frac{1}{2}\delta(f) + \frac{e^{-j2\pi f10}}{j2\pi f}$

$u(-t) \Longleftrightarrow U(-f) = \frac{1}{2}\delta(f) - \frac{1}{j2\pi f}$



$\hookrightarrow$ call this $v(t) \Leftrightarrow V(f) = U(-f)$

$v(t+10) \Leftrightarrow V(f)e^{+j2\pi f10} = \frac{1}{2}\delta(f) - \frac{e^{+j2\pi f10}}{j2\pi f}$

$\Rightarrow X(f) = \frac{1}{2}\delta(f) + \frac{e^{-j2\pi f10}}{j2\pi f} + \frac{1}{2}\delta(f) - \frac{e^{+j2\pi f10}}{j2\pi f}$

$= \delta(f) - \frac{20}{20}\frac{\sin 20\pi f}{\pi f}$

$= \delta(f) - 20\,\mathrm{sinc}\,20f$

$\Rightarrow 4x(t) = 4u(t-10) + 4u(-t-10)$

$\updownarrow$

$4\delta(f) - 80\,\mathrm{sinc}\,20f$

ANS