ECEN4503 Quiz #9 15 April 2011 Name Solution

1) A 4 watt white noise waveform x(t) has a PDF $f_X(x) = 0.5\delta(x-\alpha) + 0.5\delta(x+\alpha)$ and autocorrelation $R_{XX}(\tau) = 4\delta(\tau)$. Added to x(t) is a signal $s(t) = 3\cos 2\pi 1000t$, yielding y(t); i.e. $y(t) = x(t) + s(t)$.

1a [5] **Sketch** $S_{YY}(\omega)$.

1b [5] Another noise waveform z(t) is formed by adding the original noise waveform to a delayed version of itself and then dividing by 2, i.e. $z(t) = [y(t) + y(t-0.001)]/2$. **Sketch** $S_{ZZ}(\omega)$.

1A) $R_{YY}(\tau) \underset{F.T.}{\Longleftrightarrow} S_{YY}(\omega)$

$$A[y(t)y(t+\tau)] = A[(x(t)+s(t))(x(t+\tau)+s(t+\tau))]$$

$$= A[x(t)x(t+\tau) + \cancel{x(t)s(t+\tau)}^{0} + \cancel{s(t)x(t+\tau)}^{0} + s(t)s(t+\tau)]$$

S.I. & BOTH Zero Mean

$$= R_{XX}(\tau) + R_{SS}(\tau)$$

$\Updownarrow$ F.T.

$$S_{XX}(\omega) + S_{SS}(\omega)$$

$$R_{SS}(\tau) \triangleq A[s(t)s(t+\tau)]$$

$$= \frac{9}{2} A[\cos(2\pi 1000\tau) + \cancel{\cos(2\pi 1000(t+\tau))}^{0}]$$

$$= 4.5 \cos 2\pi 1000\tau$$

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1B) $R_{ZZ}(\tau) \underset{F.T.}{\Longleftrightarrow} S_{ZZ}(\omega)$ $\quad \Delta \triangleq 0.001$

$$A[z(t)z(t+\tau)] = \frac{1}{4} A[(y(t)+y(t-\Delta))(y(t+\tau)+y(t+\tau-\Delta))]$$

$$= \frac{1}{4} A[y(t)y(t+\tau) + y(t)y(t+\tau-\Delta) + y(t-\Delta)y(t+\tau) + y(t-\Delta)y(t+\tau-\Delta)]$$

$$= \frac{1}{4}[R_{YY}(\tau) + R_{YY}(\tau-\Delta) + R_{YY}(\tau+\Delta) + R_{YY}(\tau)]$$

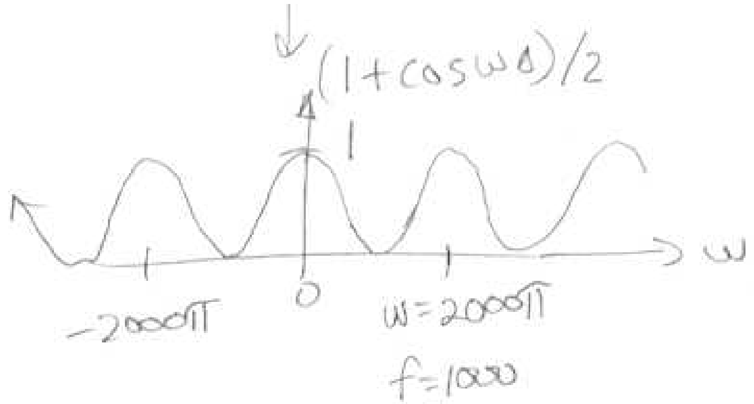
$\Updownarrow$ F.T.

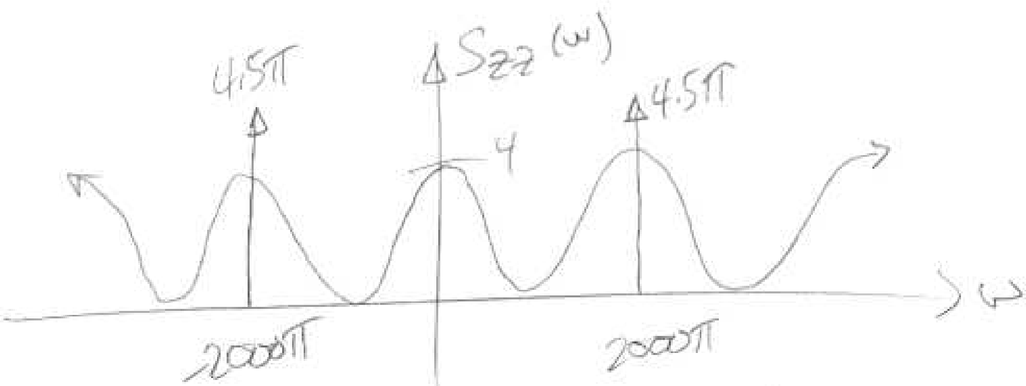
$$= \frac{1}{4}\left[2S_{YY}(\omega) + S_{YY}(\omega)e^{-j\omega\Delta} + S_{YY}e^{j\omega\Delta}\right]$$

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$$

$$= \frac{1}{4}\left[2S_{YY}(\omega) + 2S_{YY}(\omega)\cos\omega\Delta\right]$$

$$= \frac{S_{YY}(\omega)}{2}\Big[\underbrace{1+\cos 2\pi f(.001)}\Big] = S_{ZZ}(\omega)$$





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