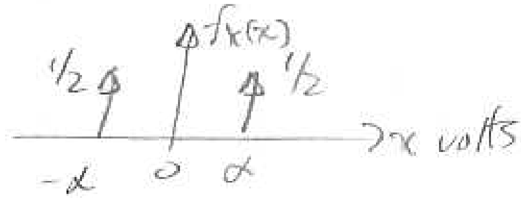
ECEN4503 Quiz #7 25 March 2011 Name Solution

1) A 4 watt noise waveform x(t) has a PDF $f_X(x) = 0.5\delta(x-\alpha) + 0.5\delta(x+\alpha)$ and autocorrelation $R_{XX}(\tau) = 4\delta(\tau)$. Another noise waveform y(t) is formed by adding the original noise waveform to ~~from~~ a delayed version of itself and then dividing by 2, i.e. $y(t) = [x(t) + x(t-0.001)]/2$.

x(t) and x(t-0.001) are known to be statistically independent.

1a) [1] **Determine** the value of α.
1b) [1] **Determine** the DC value of x(t)
1c) [1] **Determine** the AC power of x(t)
1d) [1] **Determine** the total power of x(t)
1e) [1] **Determine** the DC value of y(t)
1f) [1] **Determine** the AC power of y(t)
1g) [1] **Determine** the total power of y(t)
1h) [2] **Write an equation** for the PDF of y(t)
1i) [1] **Write an equation** for the autocorrelation of y(t).



$E[X^2] = 4 = \alpha^2 \frac{1}{2} + (-\alpha)^2 \frac{1}{2}$
$= \alpha^2$
$\Rightarrow \alpha = \underline{\underline{\pm 2}}_{ANS}$

1b) $E[X] = \underline{\underline{0\ v}}_{ANS}$ By Inspection

1c) $\sigma_X^2 = E[X^2] - E[X]^2 = \underline{\underline{4\ \text{watts}}}_{ANS}$

1d) $E[X^2] = 4\ \underline{\underline{\text{watts}}}_{ANS}$ per problem statement

$\Delta = 0.001$

1e) $A[y(t)] = A\left[\frac{x(t) + x(t-\Delta)}{2}\right] = \underline{\underline{0\ \text{vdc}}}_{ANS}$

1g) $A[y(t)^2] = \frac{A[x(t)^2 + 2x(t)x(t-\Delta) + x(t-\Delta)^2]}{4}$ (Note $x(t)$ & $x(t-\Delta)$ are S.I. & zero mean. Also $A[x(t)] = A[x(t-\Delta)]$)

$= \frac{2}{4} A[x(t)^2]$

$= \underline{\underline{2\ \text{watts}}}_{ANS}$

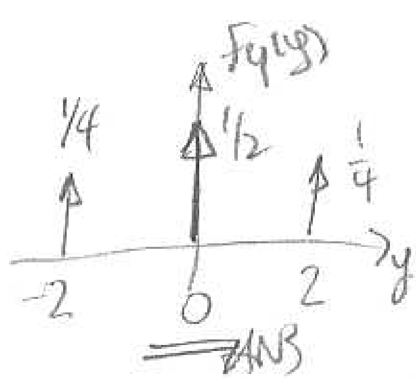
1f) $\sigma_Y^2 = A[y(t)^2] - A[y(t)]^2 = \underline{\underline{2\ \text{watts}}}_{ANS}$
[Note that the noise power is reduced.]

1h)

$x(t)$	$x(t-0.001)$	$y(t)$
2	2	2
2	-2	0
-2	2	0
-2	-2	-2

Note: 4 possible equally likely combos.

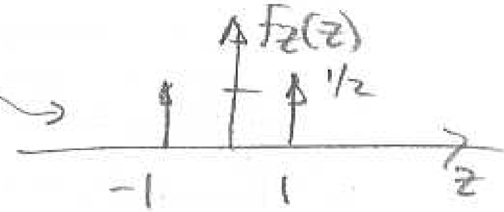
$f_Y(y) = \frac{\delta(y)}{2} + \frac{\delta(y+2)}{4} + \frac{\delta(y-2)}{4}$



Or $y(t) = \frac{x(t)}{2} + \frac{x(t-\Delta)}{2}$; $x(t)$ & $x(t-\Delta)$ have identical PDFs

Let $z(t) = \frac{x(t)}{2}$

then $z(t)$ & $z(t-\Delta)$ have identical PDFs



(Mapping of random voltage X to random voltage Z via $Z = \frac{X}{2}$)

$f_Y(y) = f_Z(z) * f_Z(z)$ (adding together 2 S.I. random voltages)

= same $f_Y(y)$ as on previous page

(i) $A[y(t)y(t+\tau)] = \frac{1}{4} A\left[\left(x(t) + x(t+\Delta)\right)\left(x(t+\tau) + x(t+\tau+\Delta)\right)\right]$

$= \frac{1}{4} A\left[x(t)x(t+\tau) + x(t)x(t+\tau+\Delta) + x(t+\Delta)x(t+\tau) + x(t+\Delta)x(t+\tau+\Delta)\right]$

$= \frac{1}{4}\left[R_{XX}(\tau) + R_{XX}(\tau+\Delta) + R_{XX}(\tau-\Delta) + R_{XX}(\tau)\right]$

$= \frac{1}{2} R_{XX}(\tau) + \frac{1}{4} R_{XX}(\tau+\Delta) + \frac{1}{4} R_{XX}(\tau-\Delta) = 2\delta(\tau) + \delta(\tau+\Delta) + \delta(\tau-\Delta)$

ANS