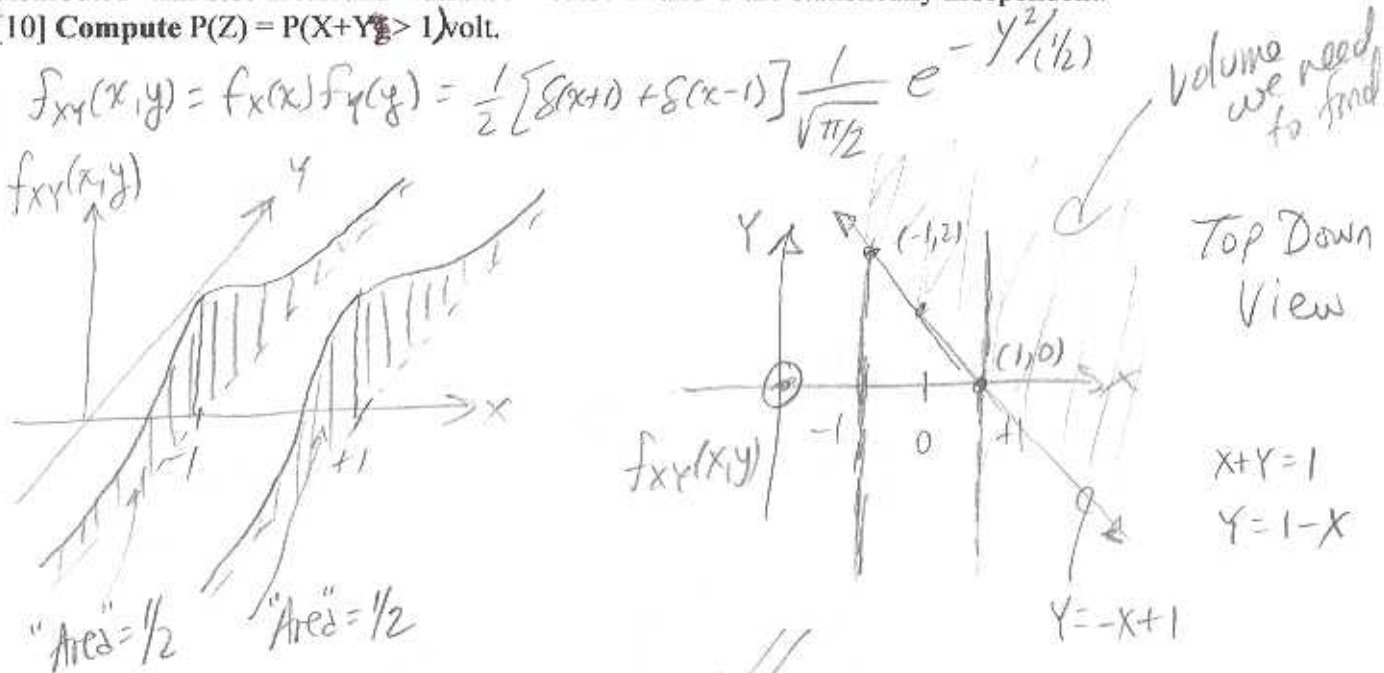


1) A serial bit stream has +1 volt pulses for Logic 1's, and -1 v pulses for Logic 0's, hence the voltage PDF  $f_X(x) = 0.5\delta(x-1) + 0.5\delta(x+1)$ . The bit stream is contaminated with additive random noise  $Y$  yielding a voltage  $Z$ , i.e.  $Z = X + Y$ . The noise is known to be Gaussian distributed with zero mean and variance = 0.25.  $X$  and  $Y$  are statistically independent.

[10] Compute  $P(Z) = P(X+Y > 1)$  volt.



$$P(X+Y > 1) = P([X = -1 \text{ AND } Y > 2] \text{ OR } [X = +1 \text{ AND } Y > 0])$$

$$= P(X = -1 \cap Y > 2) + P(X = +1 \cap Y > 0)$$

Note: [ ] events are mutually Exclusive  
 $\Rightarrow$  Don't need to subtract any terms

$$= P(X = -1)P(Y > 2 | X = -1) + P(X = +1)P(Y > 0 | X = +1)$$

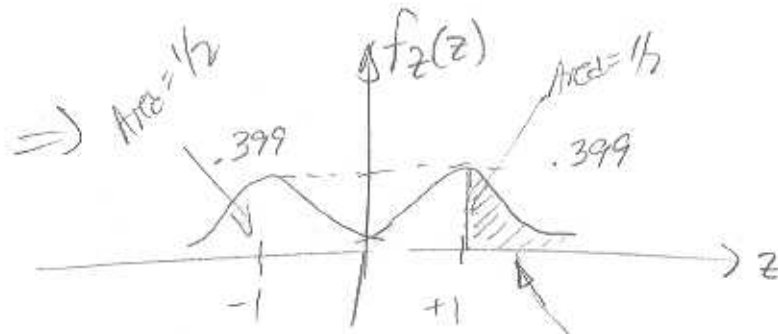
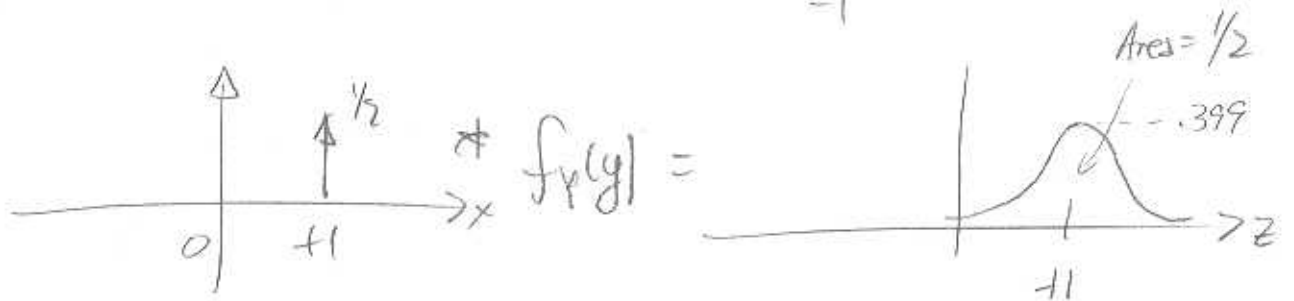
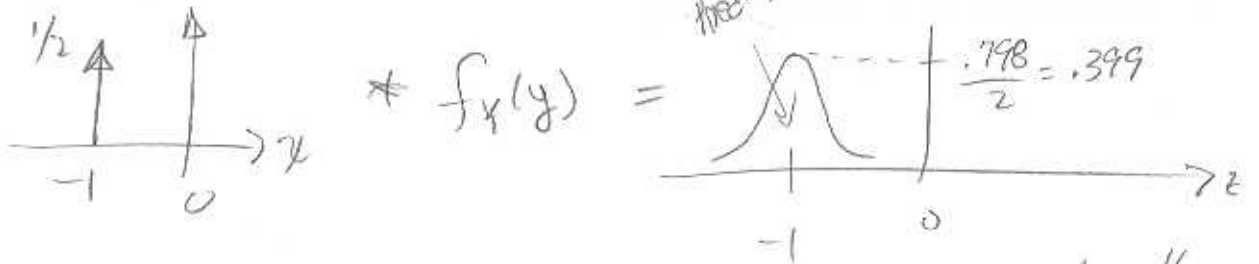
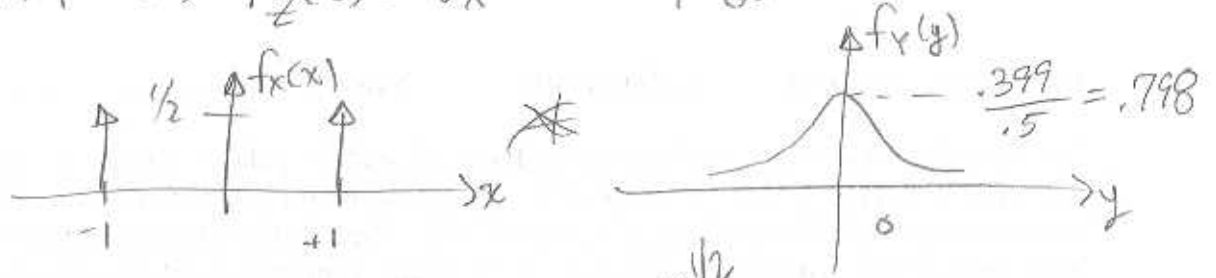
$$= \frac{1}{2} \int_2^{\infty} \frac{1}{\sqrt{\pi/2}} e^{-2y^2} dy + \frac{1}{2} \left(\frac{1}{2}\right)$$

$$= \frac{1}{2} \left(1 - F\left(\frac{2-0}{0.5}\right)\right) + \frac{1}{4} = 0 + \frac{1}{4} = \frac{1}{4}$$

ANS

OR

$$Z = X + Y \Rightarrow f_z(z) = f_x(x) * f_y(y)$$



$P(Z > 1) = \text{Shaded Area} + \text{Right hand tail of left Bell Shaped Curve that's } > +1$

Symmetry  $\Rightarrow$   
 $\frac{1}{2}$  of Area of Right Bell Shaped Curve is  $> 1$

$$\frac{1}{2} \left[ 1 - F\left(\frac{+1 - (-1)}{.5}\right) \right]$$

$$\Rightarrow \text{Probability} = \frac{1}{4} + \frac{1}{2} [1 - F(4)]$$

$$= \frac{1}{4} + \frac{1}{2} [1 - 1] = \frac{1}{4} \text{ ANS}$$