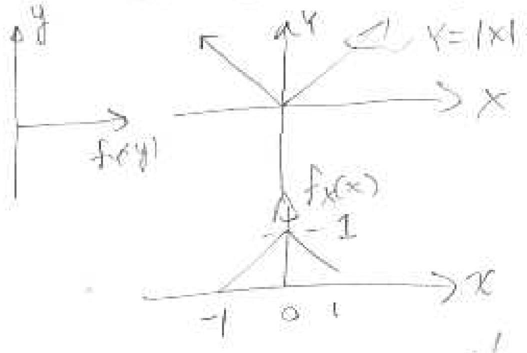
ECEN4503 Quiz #5 25 February 2011 Name Solution

1) A zero mean random voltage X is know to have PDF $f_X(x) = 1 - |X|;\ -1 \le X \le 1$. This voltage is input to a full wave rectifier, yielding an output voltage $Y = |X|$.

1a [3] **Sketch** $f_Y(y)$.

1b [2] **Compute** the DC output voltage E[Y].

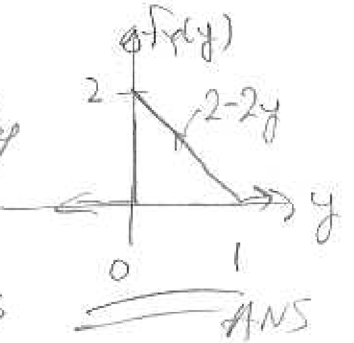


Interval $0 \le X \le 1 \Rightarrow f_Y(y) = \frac{1-|X|}{1}\Big|_{y=x}$

$= 1-|y|$

Interval $-1 \le X < 0 \Rightarrow f_Y(y) = 1 - \frac{|X|}{1}\Big|_{y=-x}$

$= 1-|y|$



$\Rightarrow f_Y(y) = 2-2y;\ |y| \le 1$

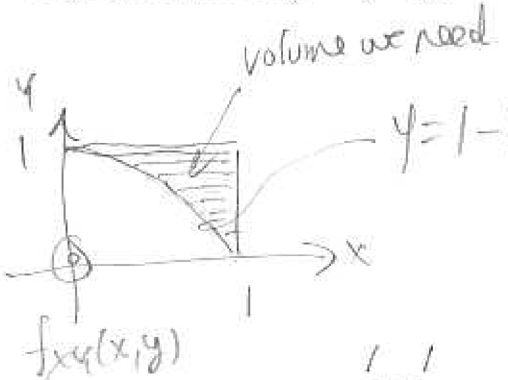
$$E[Y] = \int_0^1 y(2-2y)\,dy = \int_0^1 (2y-2y^2)\,dy = y^2 - \frac{2}{3}y^3\Big|_0^1 = 1 - \frac{2}{3} = \frac{1}{3} \text{ ANS}$$

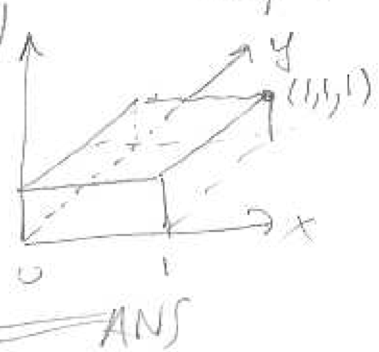
2) Computer random number generators typically output statistically independent numbers that are uniformly distributed over the interval [0,1]. Define a random variable X to be one computer generated random number, and a random variable Y to be a second computer generated random number.

2a) [2] **Sketch** $f_{XY}(x,y)$.

2b) [3] **Compute** $P(x^2 + y > 1)$.

$$f_{XY}(x,y) = f_X(x) f_Y(y) = 1 \cdot 1 = 1;\quad 0 \le x \le 1,\ 0 \le y \le 1$$





$$P(x^2+y>1) = \int_0^1\int_{\sqrt{1-y}}^1 1\,dx\,dy = \int_0^1 x\Big|_{\sqrt{1-y}}^1 dy = \int_0^1 \left(1-\sqrt{1-y}\right)dy$$

$$= y + \frac{2}{3}(1-y)^{3/2}\Big|_0^1 = (1+0) - (0 + 2/3) = 1/3 \text{ ANS}$$