

1) Roulette is a casino game in which a roulette wheel is spun in one direction and a ball is rolled in the other. Eventually the wheel and ball slow, and the ball falls at random into one of the 38 pockets on the wheel. 18 of these wheels are black, 18 red, and two green.

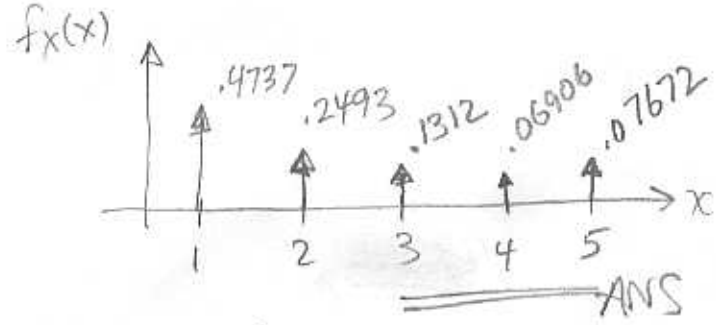
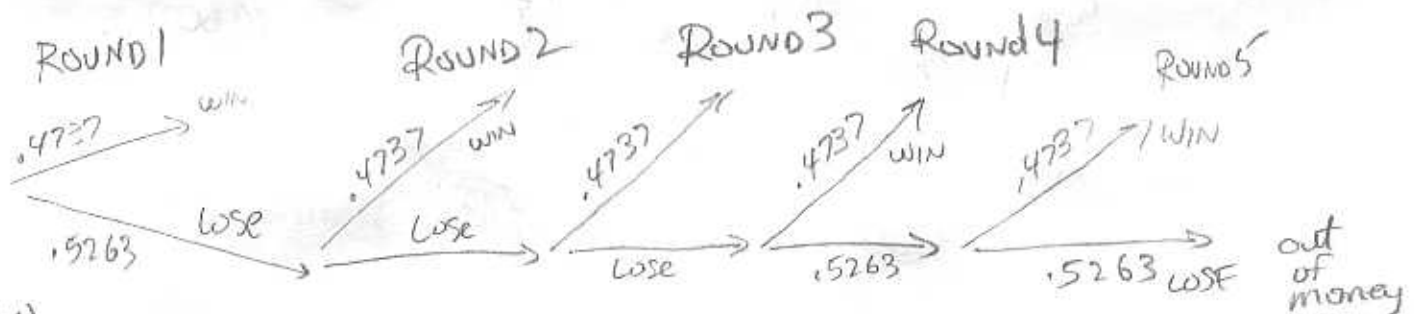
Your friend Jean has a "fool proof" plan to win at roulette. Jean has \$3100 in cash on hand and is going to make an initial bet of \$100 on black. In the event of a loss, Jean will double the bet each play of the game (so long as the cash on hand hasn't been exhausted). In the event of a win at any time, Jean will walk away after receiving the payoff. The payoff for a winning bet on black is equal to double the amount bet. For example a \$400 bet on black will yield \$800 if the ball ends up in a black pocket and \$0 if the ball ends up in a red or green pocket.

[2] Compute P(a bet on black wins).

[3] Define a random variable X to be the number of times Jean might end up playing the game. In this scenario, X is an integer number between 1 and 5. Sketch $f_X(x)$.

[5] Define a random variable Y to be the profit Jean will walk away with, profit being defined as (\$ winnings) - (\$ bets placed). Compute E[Y].

$P(\text{win}) = \frac{18}{38} = .4737$ ANS



Check

.	.4737
.	.2493
.	.1312
.	.06906
+	.07672
	<hr/>
	.99998

$P(X=2) = P(\text{Lose Round 1} \cap \text{Win Round 2})$
 $= P(\text{Lose Round 1}) P(\text{Win Round 2} | \text{Lose 1})$
 $= .5263 (.4737) = .2493$
 $P(X=3) = .5263^2 .4737 = .1312$
 $P(X=4) = .5263^3 .4737 = .06906$
 $P(X=5) = .5263^4 = .07672$

IF plays one round \Rightarrow Bet \$100, Win \$200, walked away
Profit = \$100

If plays exactly 2 rounds: Bet $\$100 + 200 = 300$ ^{\\$}
 Won 400 ^{\\$}, walked away
 \Rightarrow profit = $\$100$

If plays exactly 3 rounds: Bet $100 + 200 + 400 = 700$ ^{\\$}
 Won 800 ^{\\$}
 \Rightarrow profit = $\$100$

If plays exactly 4 rounds: Bet $100 + 200 + 400 + 800 = 1500$ ^{\\$}
 Won 1600 ^{\\$}
 \Rightarrow profit = $\$100$

If plays exactly 5 rounds: Bet $100 + 200 + 400 + 800 + 1600 = \3100

profit = $\$100$ \leftarrow 47.37% chance won $\$3200$
 profit = $-\$3100$ \leftarrow 52.62% chance won $\$0$

$$E[Y] = \$100 (.4737 + .2493 + .1312 + .06906) + 100 (.5263)^4 .4737 - 3100 (.5263)^5$$

$$= \$92.33 + 3.63 - 125.18 = \underline{\underline{-\$29.22}} \text{ ANS}$$

Note that the odds you turn a $\$100$ profit

$$= 1 - P(\text{lose 5 straight games}) = 1 - .5263^5$$

