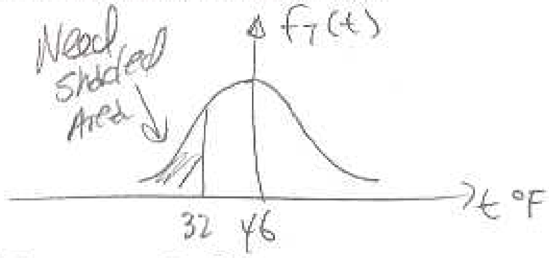
ECEN4503 Quiz #3 10 February 2010 Name Solution

1) The peak temperature T in degrees Fahrenheit on January 15th in Tulsa is a Gaussian random variable with a standard deviation of 10 degrees. With probability 0.5, the peak temperature T exceeds 46 degrees.

1a) [3] **Compute** P(peak temperature is < 32 degrees Fahrenheit on January 15).

1b) [2] **Compute** P(peak temperature is < 32 degrees Fahrenheit for 5 consecutive January 15's).

1a) $P(T<32) = \int_{-\infty}^{32} \frac{1}{\sqrt{2\pi 100}} e^{-(t-46)^2/200}\, dt$



$= F\left(\frac{32-46}{10}\right) = F(-1.4)$

$= 1 - F(1.4)$ (equation B-4) $= 1 - .9192 = \underline{0.0808}$ ANS

1b) Define $X \triangleq$ # of times in 5 trials temp is < 32° on JAN 15

$\Rightarrow X$ is Binomial (assuming S.I. trials)

$P(X=5) = \binom{5}{5} .0808^5 .9192^0 = .0808^5 = \underline{3.444(10^{-6})}$ ANS

2) A random variable X has PDF $f_X(x) = \alpha x^2 + \beta x;\ 0 \le x \le 1$, where α and β are positive constants.

2a) [2] What conditions on α and β are necessary to insure $f_X(x)$ is a valid PDF?

2b) [3] **Compute** the second moment when $\alpha = 1$.

2A) To be a PDF $\int_0^1 (\alpha x^2 + \beta x)\,dx \overset{\text{must}}{=} 1$

$\Rightarrow \left.\frac{\alpha x^3}{3} + \frac{\beta x^2}{2}\right|_0^1 = \frac{\alpha}{3} + \frac{\beta}{2} \overset{\text{must}}{=} 1$

or $\underline{2\alpha + 3\beta \overset{\text{must}}{=} 6}$ ANS

2B) $\alpha = 1 \Rightarrow \beta = 4/3$

$E[X^2] = \int_0^1 x^2\left(x^2 + \frac{4}{3}x\right)dx = \int_0^1 \left(x^4 + \frac{4}{3}x^3\right)dx = \left.\frac{x^5}{5} + \frac{x^4}{3}\right|_0^1$

$= 1/5 + 1/3 = 8/15 = \underline{.5333}$ ANS