

1) Suppose the $P(\text{a mother has a boy}) = P(\text{a mother has a girl}) = 0.5$ and that in subsequent births the sex is statistically independent. You randomly pick a family with two children.

1a) [2] Compute $P(\text{this family has 1 boy and 1 girl})$.

1b) [1] Compute $P(\text{the second child is a boy} \mid \text{first child is a girl})$.

1c) [2] Compute $P(\text{the other child is a boy} \mid \text{one child is a girl})$.

Two children \Rightarrow 4 equally likely combos:

| | | | | |
|---|-----------------|-----------------|-----|-----|
| | 1 ST | 2 ND | B | G |
| B | | | .25 | .25 |
| G | | | .25 | .25 |

1A) $P(1 \text{ BOY} \cap 1 \text{ GIRL}) = 2/4 = \underline{\underline{1/2}}$ ANS

1B) $P(\# 2 \text{ is B} \mid \# 1 \text{ is G}) = \underline{\underline{1/2}}$ ANS

1C) $P(\text{other is B} \mid \text{one is a girl}) = \underline{\underline{2/3}}$ ANS

Comment! # boys impacts # of girls \Rightarrow #'s are NOT S.I.

2) Suppose that when a baseball player gets a hit and safely reaches base, a 1 base hit is twice as likely as a 2 base hit, which is twice as likely as a 3 base hit, which is twice as likely as a 4 base hit. The probability the player gets a hit is 0.3, and the probability the player makes an out (equivalent to a 0 base hit) is 0.7. Define a random variable X to be the number of bases the player reaches.

[5] Sketch $f_X(x)$.

| | |
|----------|--------------------------------|
| $4B = 1$ | $P(4B \mid \text{HIT}) = 1/15$ |
| $3B = 2$ | $P(3B \mid \text{HIT}) = 2/15$ |
| $2B = 4$ | $P(2B \mid \text{HIT}) = 4/15$ |
| $1B = 8$ | $P(1B \mid \text{HIT}) = 8/15$ |

$\Rightarrow P(1B \cdot \text{HIT}) = P(\text{HIT}) P(1B \mid \text{HIT})$
 $= .3(8/15) = .24/15 = .016$

$P(2B \cdot \text{HIT}) = \dots = .3(4/15) = .08$

$P(3B \cdot \text{HIT}) = .04$

$P(4B \cdot \text{HIT}) = .02$

$\underline{\underline{\Sigma = 0.30}}$

