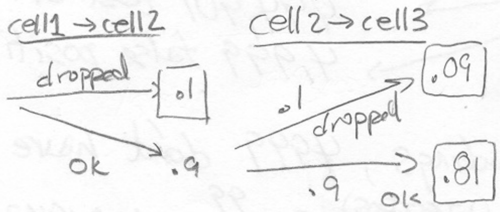
ECEN4503 Quiz #1 22 January 2010 Name Solution

1) During a cellular phone call, when a user moves from one cell to another adjacent cell a *handoff* occurs. Suppose 90% of the time a handoff succeeds and the user can continue their call, and 10% of the time the call is abruptly dropped (i.e. terminated) as the new cell has no spare resources to support the handoff. Added: Note a specific call can only be dropped once.

[2] A user successfully takes a call in cell #1. If the user then moves to adjacent cell #2, what is the probability this call will be dropped?

[3] A user successfully takes a call in cell #1. It the user then moves to adjacent cell #2, and then to adjacent cell #3, what is the probability this call will be dropped?

From problem statement, this probability is $\underline{\underline{0.1}}$ ANS



$P(\text{dropped}) = 1 - P(\text{OK after 2 handoffs})$

$= 1 - .81 = \underline{\underline{0.19}}$ ANS

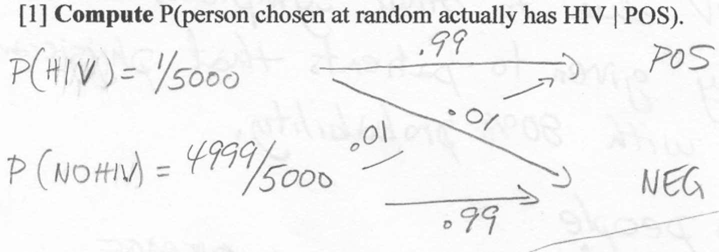
2) Suppose that in the general population 1 in 5000 people carry the HIV virus. A test for the HIV virus yields either a positive (event POS) or negative (event NEG) result. Suppose the test gives the correct answer 99% of the time.

[1] **Compute** P(POS), the probability a person chosen at random tests positive for HIV.

[2] **Compute** P(NEG), the probability a person chosen at random tests negative for HIV.

[1] **Compute** P(NEG | person has HIV).

[1] **Compute** P(person chosen at random actually has HIV | POS).



Total Probability

$P(POS) = \frac{1}{5000}(.99) + \frac{4999(.01)}{5000} = \frac{.99 + 49.99}{5000} = \underline{\underline{0.010196}}$ ANS

$P(NEG) = 1 - P(POS) = \underline{\underline{.989804}}$ ANS

$P(NEG \mid \text{Person has HIV}) = \underline{\underline{0.01}}$ ANS from problem statement

$P(POS)\,P(HIV \mid POS) = P(HIV)\,P(POS \mid HIV)$ Bayes theorem

$\Rightarrow P(HIV \mid POS) = \frac{P(HIV)\,P(POS \mid HIV)}{P(POS)} = \frac{(1/5000)(.99)}{.010196} = \underline{\underline{.01942}}$ ANS

Note this means that if you were randomly chosen for an HIV test, & you test positive, you probably don't have HIV! ⇒ Random testing is ineffective.*

Example) Suppose have 500,000 people (* Given these probabilities)

On average

100 have HIV $\xrightarrow{99\%}$ ON AVERAGE 99 test positive

→ 1 false negative

499,900 don't $\xrightarrow{99\%}$ 494,901 test OK

→ 4,999 false positives

⇒ of 5098 positive readings, 4999 don't have HIV!

$$P(\text{Have HIV} \mid \text{Test POS}) = \frac{P(\text{HAVE HIV} \cap \text{TEST POS})}{P(\text{Test POS})} = \frac{99}{5098} = 0.01942$$

Why bother with this test in 1st place? Normally a physician wouldn't order one unless they suspected a patient has HIV due to other symptoms or behavior.

Suppose test is only given to patients that physician suspects has HIV with 80% probability.

EX) 500,000 people

ON AVERAGE

400,000 HAVE HIV $\xrightarrow{99\%}$ ON AVERAGE 396,000 test positive

→ 4,000 false negative

100,000 DON'T $\xrightarrow{99\%}$ 99,000 Test OK

1,000 false positives

Under these circumstances

$$P(\text{Have HIV} \mid \text{Test Positive}) = \frac{396{,}000}{397{,}000} = 0.9975 \text{ ANS}$$

⇒ Targeted testing is effective.