**ECEN4503 Quiz #1 19 January 2011 Name** Solution

1) The outcomes of an experiment are integers between 10 and 20, inclusive. Thus the sample space S = {10, 11, 12, ... , 20}. The probability of any outcome is proportional to the inverse of that number and is equal to 1.301/(number). For example, P(outcome = 10) = 1.301/10 = 0.1301, P(outcome = 11) = 1.301/11 = 0.1183, etc. Define the event A = {outcome is 11, 13, 17, or 19} and B = {outcome is exactly divisible by 6}.

1a) [1] Compute P( A )

1b) [2] Compute P( A ∪ B )

1c) [2] Compute P( A ∩ B )

1d) [2] Compute P( A | B )

1A) P(A) = P(11 or 13 or 17 or 19)
= P(11) + P(13) + P(17) + P(19) − 0 (Mutually Exclusive)
= .1183 + .1001 + .07653 + .06847
= .3634 ANS

1B) P(B) = P(12 or 18) = P(12) + P(18) − 0
= .1084 + .07228 = .1807

P(A∪B) = P(A) + P(B) − P(AB) (= 0)
= .5441 ANS

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{.5441} = 0 \text{ ANS}$$

1c) P(A ∩ B) = 0 ANS since no elements are in both sets.

2) [3] A digital chip contains 100 transistors and 400 connections. The probability of a faulty transistor is 0.001, and the probability of a faulty connection is 0.0001. What is the probability that a chip taken at random is defective, assuming all defects are statistically independent? Note that the chip is considered to be defective if it has ≥ one fault of any kind.

Event A ≜ all transistors good ⇒ $P(A) = .999^{100} = .9048$

Event B ≜ all connections good ⇒ $P(B) = .9999^{400} = .9608$

P(chip is good) = P(A ∩ B) = P(A)P(B) (S.I.) = .9048 (.9608) = .8693

P(chip is bad) = 1 − P(chip is good) = .1307 ANS

Note: P(A) = P(transistor #1 good ∩ #2 is good ∩ ... ∩ #100 is good)
(S.I.) = P(#1 good) P(#2 good) ... P(#100 is good) = $P(\text{good})^{100}$

2) Alternatively

Event A = $\geq 1$ transistor bad  $P(A) = 1 - P(\text{all good}) = .0952$

Event B = $\geq 1$ connection bad  $P(b) = 1 - P(\text{all good}) = .0392$

Event C $\triangleq$ Chip is Bad

= A or B = A+B

$\Rightarrow P(C) = P(A+B)$

$P(C) = P(A) + P(B) - P(AB)$ ) S.I.

$= P(A) + P(B) - P(A)P(B)$

$= .0952 + .0392 - (.0952)(.0392) = .1344 - .00373$

$= .1307$ ANS