

**ECEN4503 Final Exam 10 December 2009**

1) Suppose over the course of time the daily temperature at Stillwater, Oklahoma in degrees Celsius is  $d(t) = 16[1 - \cos(\omega_0 t)] + 4n(t)$ , where  $n(t)$  is an S.I. Gaussian noise process with autocorrelation  $R_{NN}(\tau) = e^{-|\alpha\tau|} \cos(2\pi\alpha\tau)$ . Here  $\omega_0 = 2\pi/31,536,000$  and  $\alpha = 1/86,400$ . Note that there are  $3600*24*365 = 31,536,000$  seconds in a 365 day year, and 1 day = 86,400 seconds.

1a) [10] **Compute** the time average mean. [Answer = 16]

1b) [15] **Write an equation** for the time average autocorrelation of  $d(t)$  in terms of  $R_{NN}(\tau)$ .

[ $256 + 128\cos\omega_0 \tau + 16R_{NN}(\tau)$ ]

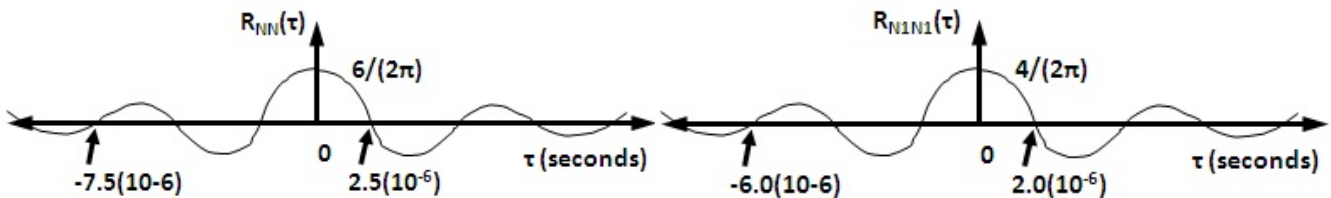
1c) [10] **Write an equation** for the power spectrum of  $n(t)$ ,  $S_{NN}(\omega)$ . [ $\alpha/(\alpha^2 + (\omega - \omega_0)^2) + \alpha/(\alpha^2 + (\omega + \omega_0)^2)$ ]

2) You are observing  $x(t)$ , a zero mean, 1 Hertz, 1 volt peak, 2 volts peak-to-peak sinusoid on an oscilloscope.

[35] If you randomly sample the voltage of this waveform, **compute** the probability the sampled voltage is  $> 0.707$  volts. [0.25]

3) The waveforms  $n(t)$  and  $n_1(t)$  are two statistically independent, band limited, White Gaussian noise waveforms with autocorrelations show below. A third waveform  $y(t)$  is formed via  $y(t) = n(t) + n_1(t)$ .

[40] **Sketch** the output power spectrum  $S_{YY}(\omega)$ .



[You should sketch the waveform

$$S_{YY}(\omega) = \begin{cases} 1.273 \cdot 10^{-6}; & -250K \leq \omega \leq -200K \\ 3.660 \cdot 10^{-6}; & -200K \leq \omega \leq 200K \\ 1.273 \cdot 10^{-6}; & 200K \leq \omega \leq 250K \end{cases}$$

4) A waveform  $y(t) = x(t) + 0.9x(t-0.8)$ . The input  $x(t)$  is a zero mean, 1 watt, band-limited white noise waveform,  $|\omega| < 5\pi$  rad/sec

4a) [20] **Sketch** the input power spectrum  $S_{XX}(\omega)$ . [ $S_{XX}(\omega) = 0.2; -5\pi \leq \omega \leq 5\pi$ ]

4b) [20] **Compute** the average power of the output  $y(t)$ . [1.81 watts]