ECEN4503 Exam #2 27 October 2009

1) A random variable $W = (\alpha X - 3Y)^2$. X and Y are statistically independent Gaussian random variables, with E[X] = 6, E[Y] = -4, $\sigma_X^2 = \sigma_Y^2 = 5$, and α = an unknown constant.

[15] **Compute** E[W]. [Answer: $41\alpha^2 + 144\alpha + 189$]

[10] **Compute** the value of α that minimizes E[W]. [-1.756]

2) A zero mean voice signal x(t) is known to have a Laplacian distributed voltage with PDF $f_X(x) = 3e^{-|6x|}$. This signal is input to a full wave rectifier yielding an output signal y(t) = |x(t)|. Hence the output voltage Y = |X|, the absolute value of the input voltage.

[10] If a volt meter is attached to x(t), what will be the volts DC reading on the meter? [0 volts]

[15] If a volt meter is attached to y(t), what will be the volts DC reading on the meter? [1/6 volt]

3) A 2nd order PDF $f_{XY}(x,y) = 1/2$; $x \geq 0$, $y \geq 0$, $x+y \leq 2$.

[25] **Write an integral equation** that could be used to evaluate P(X+Y > 1). *Do not bother evaluating the integral.*

Note: The correct answer can be obtained pretty much by inspection, but the purpose of this problem is to see if you can set up an integral equation that could then be evaluated by math software, so give me an integral equation!

[There are at least three ways....]

$$1 - \int_0^1 \int_0^{1-y} \frac{1}{2}\, dx\, dy = 0.75$$

$$\int_1^2 \int_0^{2-y} \frac{1}{2}\, dx\, dy + \int_0^1 \int_{1-y}^{2-y} \frac{1}{2}\, dx\, dy = 0.75$$

$$\int_0^1 \int_{1-x}^{2-x} \frac{1}{2}\, dy\, dx + \int_1^2 \int_0^{2-x} \frac{1}{2}\, dy\, dx = 0.75$$

4) A random noise voltage X is known to be uniformly distributed over the interval [-1,1]. A random binary square wave voltage Y is known to have PDF $f_Y(y) = 0.5[\delta(y+2) + \delta(y-2)]$. A random voltage Z = X + Y. X & Y are known to be S.I.

[25] **Sketch** $f_Z(z)$. [You should sketch two pulses, each of height 1/4 and width 2. One pulse is centered at z = -2, the other at z = +2]