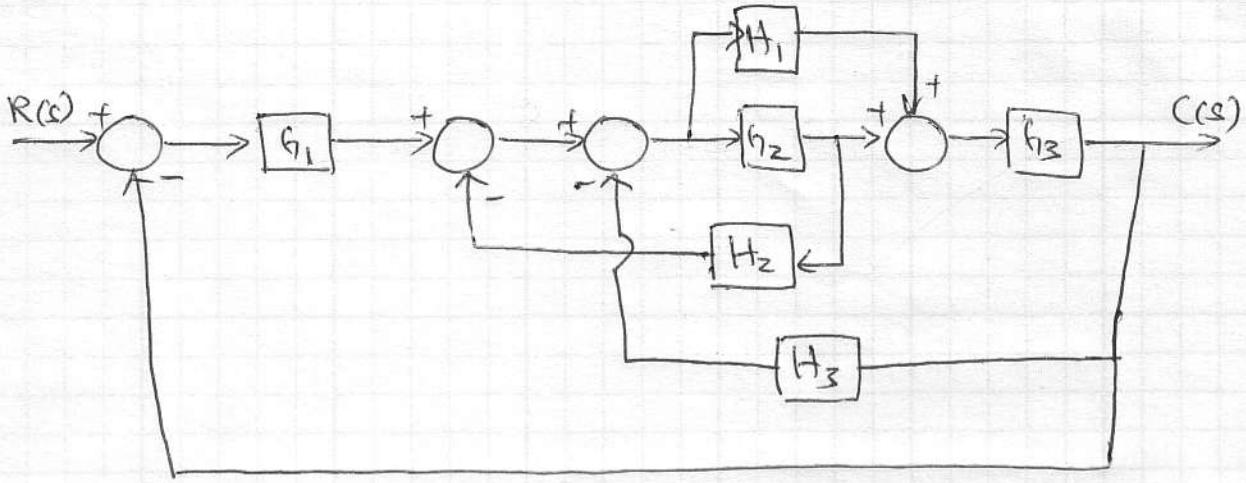
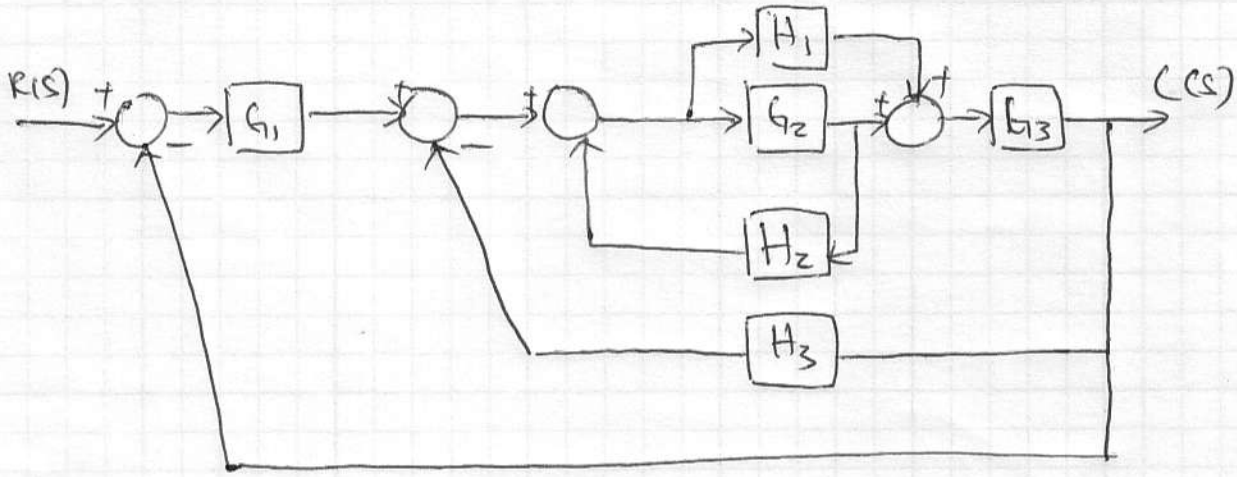
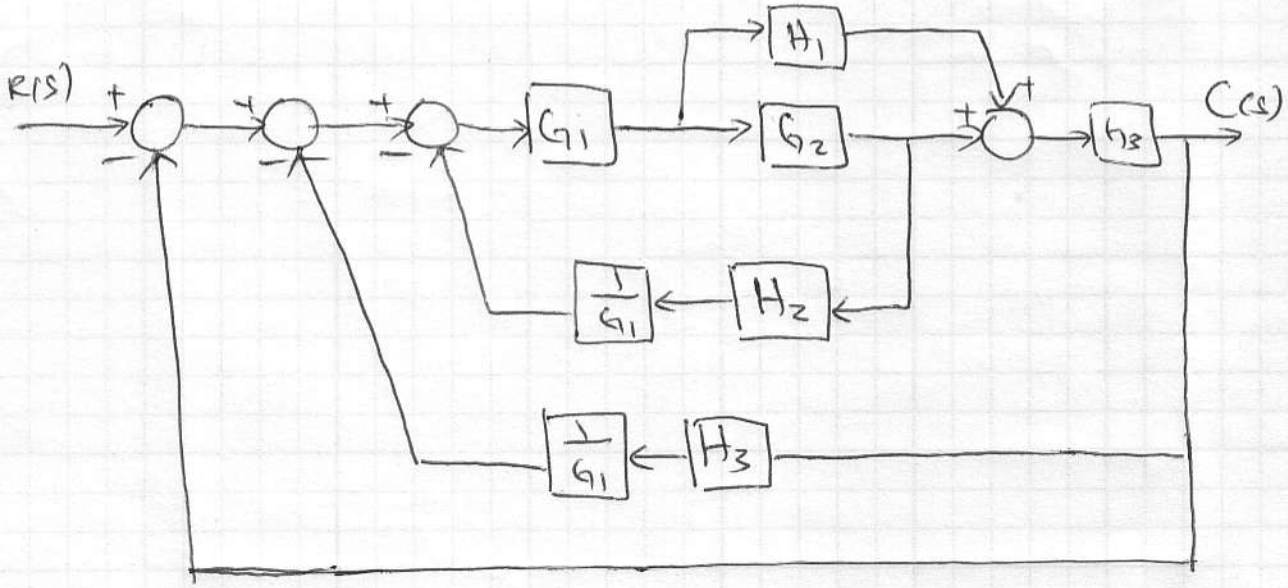
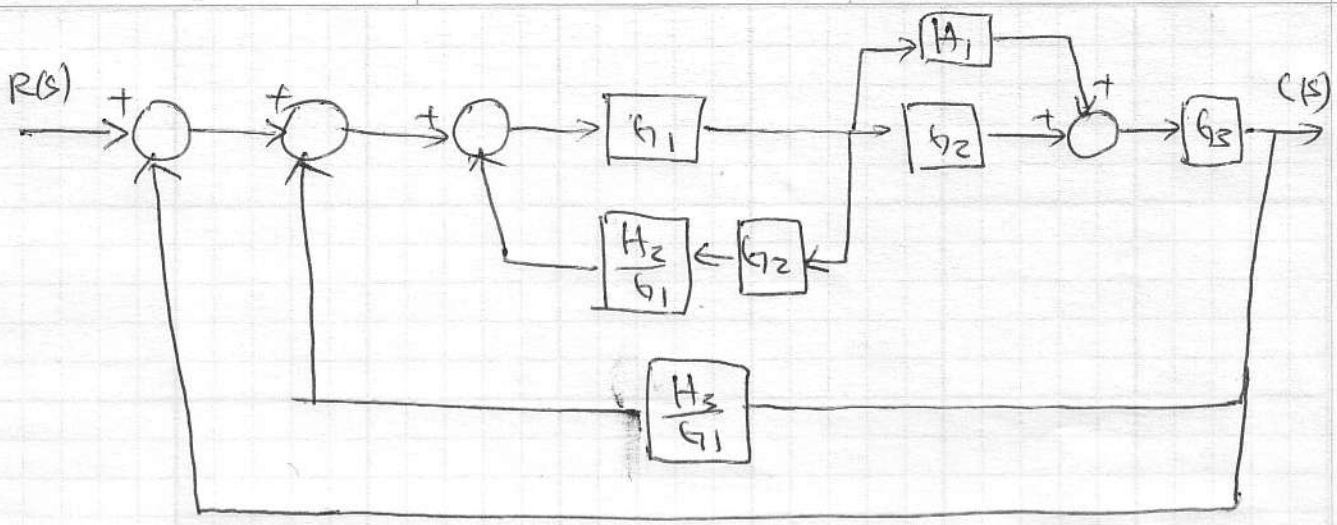
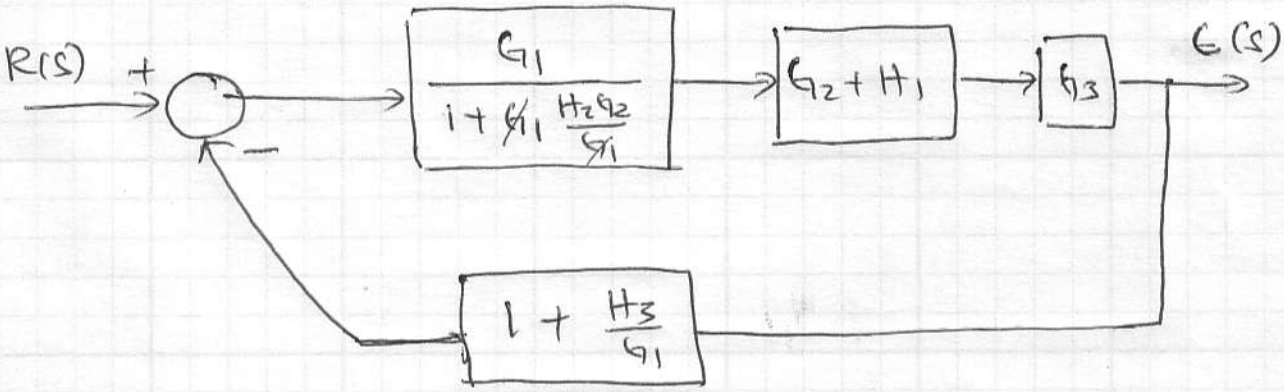
Problem 1


R(s)
G1
H1
G2
H2
G3
C(s)
H3


R(s)
G1
H1
G2
H2
G3
C(s)
H3


R(s)
G1
H1
G2
G3
C(s)
1/G1
H2
1/G1
H3

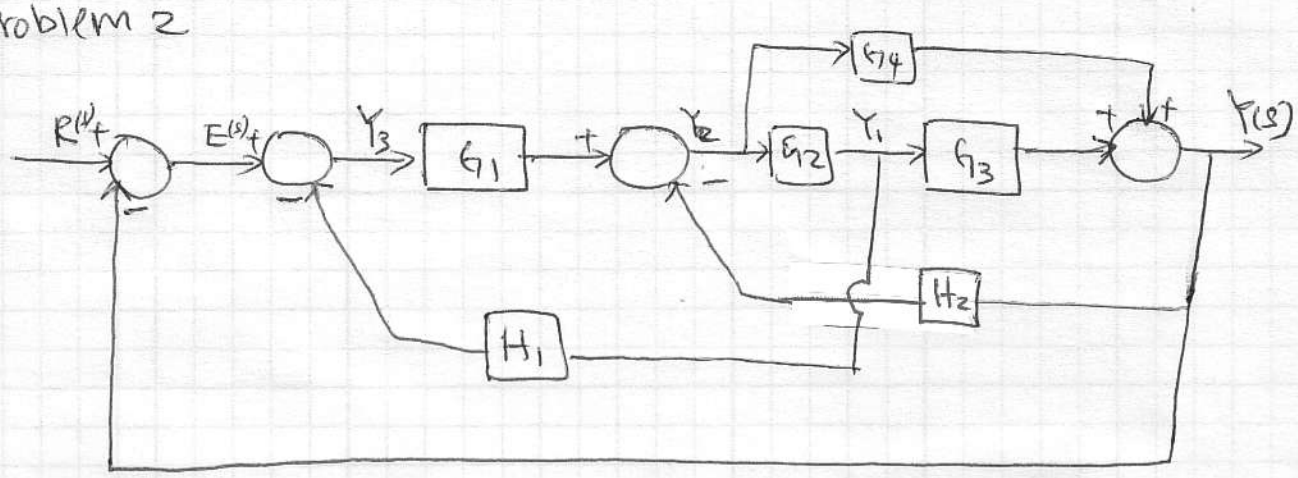


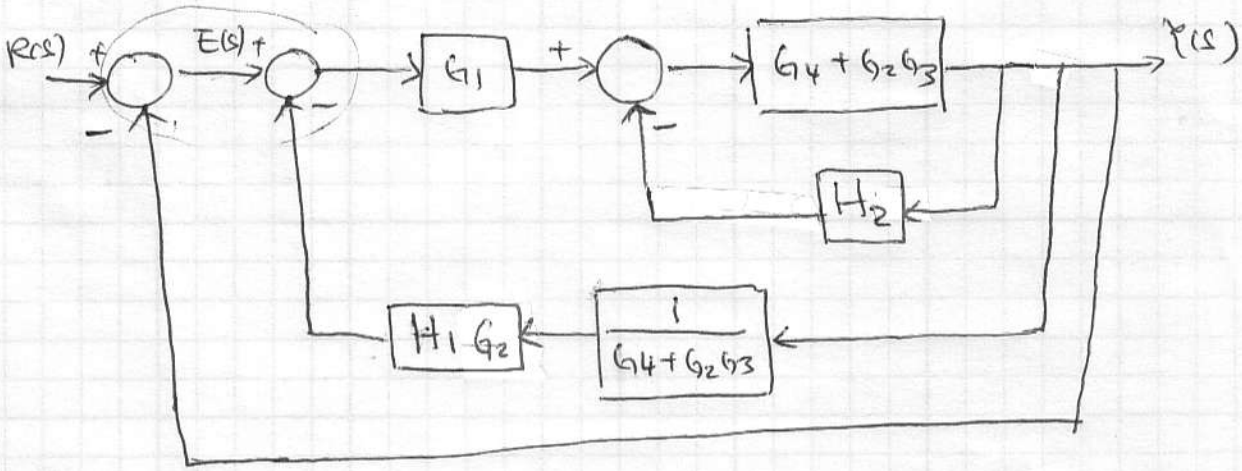


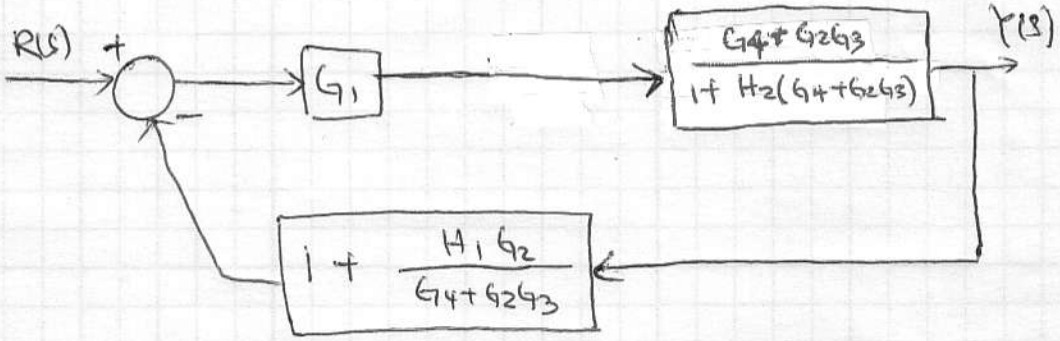
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1 G_3 (G_2 + H_1)}{1 + H_2 G_2}}{1 + \left(\dfrac{G_1 G_3 (G_2 + H_1)}{1 + G_2 H_2}\right)\left(\dfrac{G_1 + H_3}{G_1}\right)}$$

$$= \frac{G_1 G_3 (H_1 + G_2)}{1 + H_2 G_2 + G_1 G_2 G_3 + G_2 G_3 H_3 + G_1 G_3 H_1 + G_3 H_1 H_3}$$

Problem 2



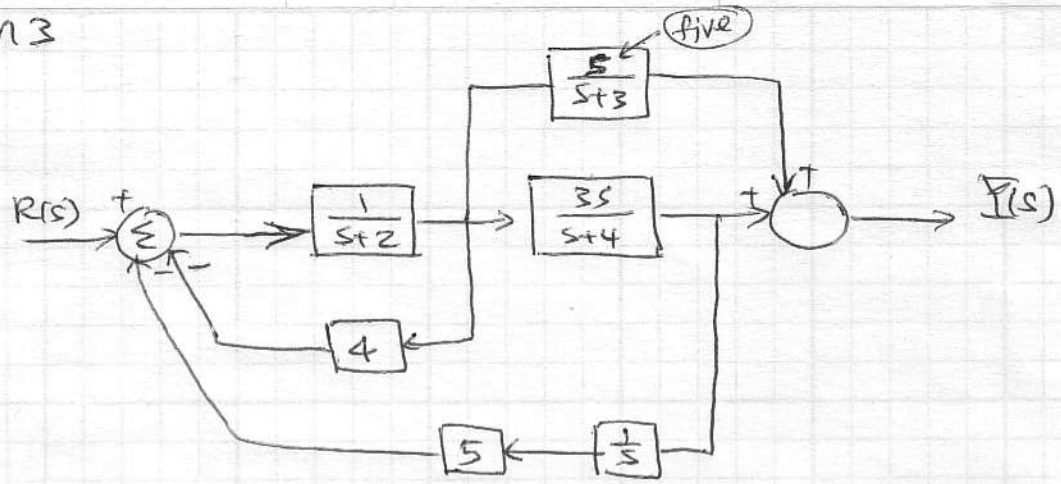


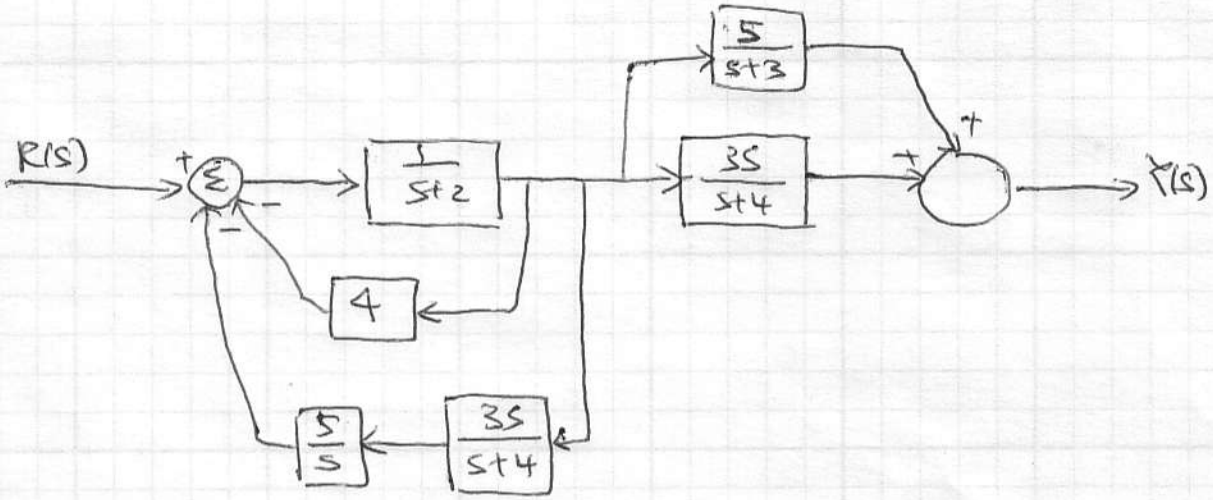


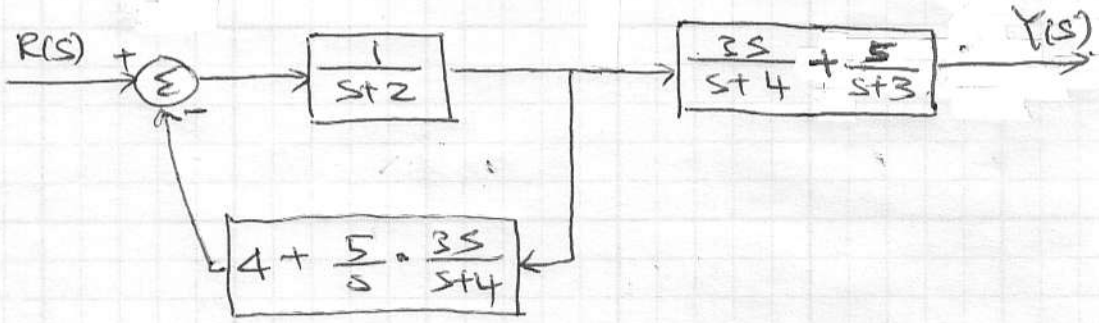
$$\frac{Y(s)}{R(s)} = \frac{G_1\left(\frac{G_4 + G_2G_3}{1 + H_2(G_4 + G_2G_3)}\right)}{1 + G_1\left(\frac{G_4 + G_2G_3}{1 + H_2(G_4 + G_2G_3)}\right)\cdot\left(1 + \frac{H_1G_2}{(G_4 + G_2G_3)}\right)}$$

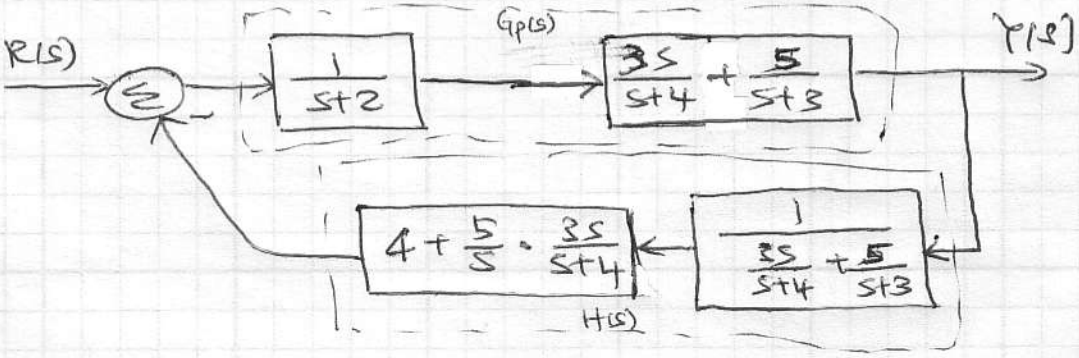
$$= \frac{G_1(G_4 + G_2G_3)}{1 + H_1G_1G_2 + H_2(G_4 + G_2G_3) + G_1(G_4 + G_2G_3)}$$

Problem 3


five
5/(s+3)
R(s)
+
−
1/(s+2)
35/(s+4)
+
+
Y(s)
4
5
1/s


5/(s+3)
R(s)
+
−
1/(s+2)
35/(s+4)
+
+
Y(s)
4
5/s
35/(s+4)


R(s)
+
−
1/(s+2)
35/(s+4) + 5/(s+3)
Y(s)
4 + 5/s · 35/(s+4)


Gp(s)
R(s)
−
1/(s+2)
35/(s+4) + 5/(s+3)
Y(s)
4 + 5/s · 35/(s+4)
1/(35/(s+4) + 5/(s+3))
H(s)

Transfer function :

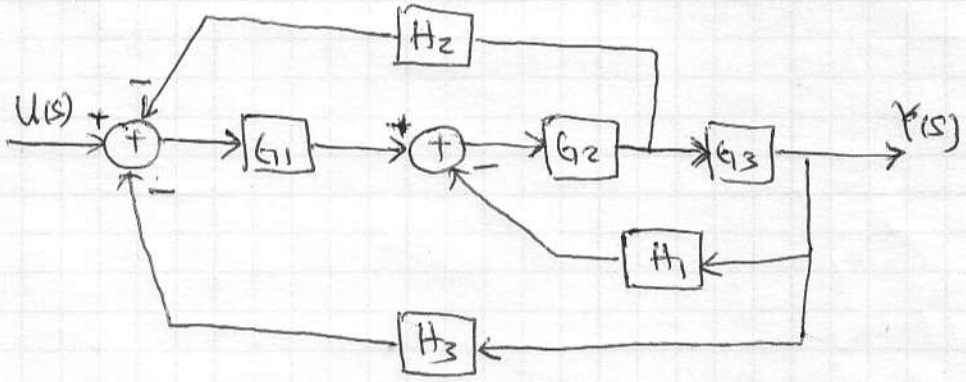
$$\frac{Y(s)}{R(s)} = \frac{G_p(s)}{1 + G_p(s)H(s)}$$

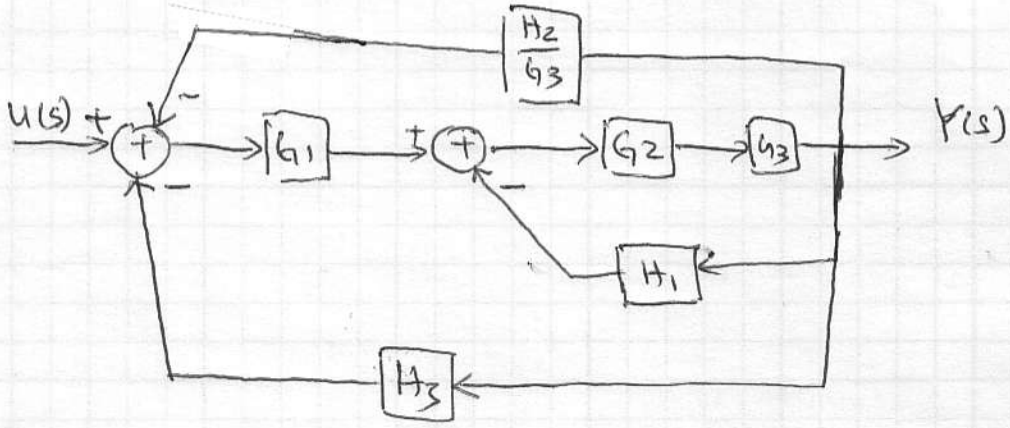
$$= \frac{\frac{1}{s+2}\left(\frac{3s}{s+4} + \frac{5}{s+3}\right)}{1 + \frac{1}{s+2}\left(\frac{3s}{s+4} + \frac{5}{s+3}\right)\left(4 + \frac{5}{s}\cdot\frac{3s}{s+4}\right)\left(\frac{1}{\frac{3s}{s+4} + \frac{5}{s+3}}\right)}$$

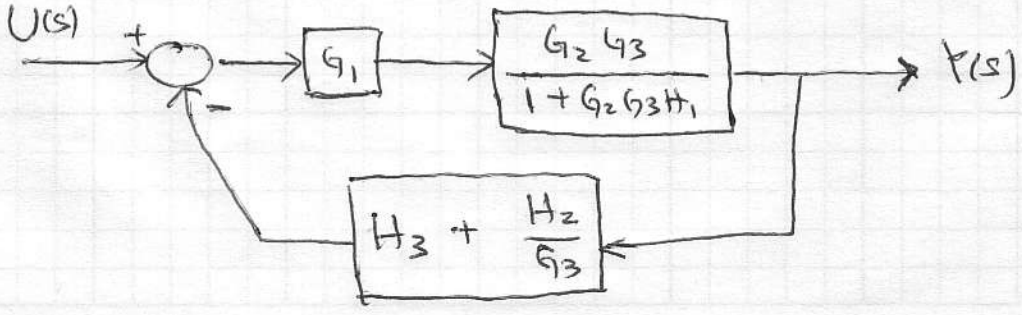
$$= \frac{5(s+4) + 3s(s+3)}{(s+2)(s+4)(s+3) + (4s+31)(s+3)}$$

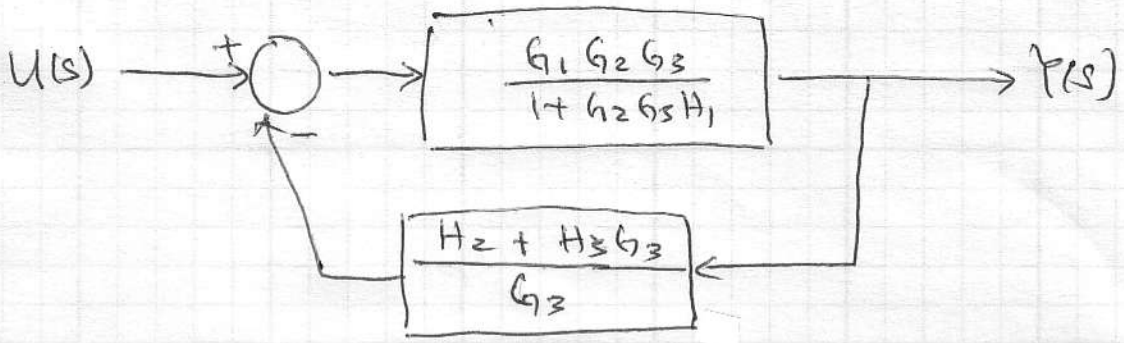
$$= \frac{3s^2 + 14s + 20}{(s+3)(s^2+10s+9)}$$

Problem 4







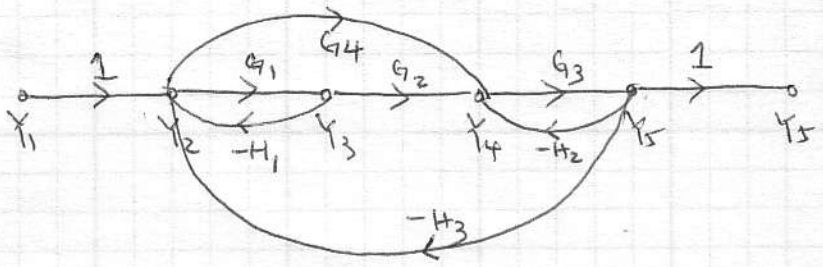


$$\frac{Y(s)}{U(s)} = \frac{\dfrac{G_1 G_2 G_3}{1 + G_2 G_3 H_1}}{1 + \dfrac{G_1 G_2 G_3}{1 + G_2 G_3 H_1} \cdot \dfrac{H_2 + H_3 G_3}{G_3}}$$

$$= \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_1 + G_1 G_2 H_2 + G_1 G_2 G_3 H_3}$$

Problem 5

(a)



Y_5 as the output

Forward paths:

$$M_1 = 1 \cdot G_1 G_2 G_3 \cdot 1 = G_1 G_2 G_3$$
$$M_2 = 1 \cdot G_4 G_3 \cdot 1 = G_4 G_3$$

Individual loops:

$$L_{11} = -G_1 H_1$$
$$L_{21} = -G_3 H_2$$
$$L_{31} = -G_1 G_2 G_3 H_3$$
$$L_{41} = -G_4 G_3 H_3$$

Non touching loop:

$$L_{12} = G_1 H_1 G_3 H_2$$

$$\Delta = 1 - (L_{11} + L_{21} + L_{31} + L_{41}) + L_{12}$$
$$= 1 + G_1 H_1 + G_3 H_2 + G_1 G_2 G_3 H_3 + G_4 G_3 H_3 + G_1 H_1 G_3 H_2$$

$$\Delta_1 = 1$$
$$\Delta_2 = 1$$

$$\frac{Y_5}{Y_1} = \frac{M_1 \Delta_1 + M_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 + G_4 G_3}{\Delta}$$

Y_4 as output

Forward paths:

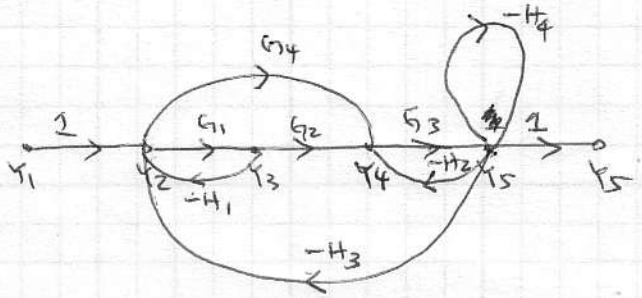
$$M_1 = G_1 G_2$$
$$M_2 = G_4$$

The loops are not changed: $L_{11}, L_{21}, L_{31}, L_{41}, L_{12}$, so Δ is unchanged. Note also, $\Delta_1 = 1$, $\Delta_2 = 1$

$$\frac{Y_4}{Y_1} = \frac{G_1 G_2 + G_4}{\Delta}$$

Note: $\Delta \rightarrow$ the denominator has not changed

b)



Y_5 as output

Forward paths:

$$M_1 = G_1 G_2 G_3$$
$$M_2 = G_4 G_3$$

Independent loops:

$$L_{11} = -G_1 H_1$$
$$L_{21} = -G_3 H_2$$
$$L_{31} = -H_4$$
$$L_{41} = -G_1 G_2 G_3 H_3$$
$$L_{51} = -G_4 G_3 H_3$$

Non_touching loops:

$$L_{12} = G_1 H_1 H_4$$
$$L_{22} = G H_1 G_3 H_2$$

$$\Delta = 1 + G H_1 + G_3 H_2 + H_4 + G_1 G_2 G_3 H_3 + G_3 G_4 H_3$$

$$\Delta_1 = 1$$
$$\Delta_2 = 1$$

$$\frac{Y_5}{Y_1} = \frac{G_1 G_2 G_3 + G_3 G_4}{\Delta}$$

Y_4 as output

$$M_1 = G_1 G_2$$
$$M_2 = G_4$$

Δ is the same since all the loops not change.

$$\Delta_1 = 1 + H_4 \quad ; \quad \Delta_2 = 1 + H_4$$

$$\frac{Y_4}{Y_1} = \frac{G_1 G_2 (1 + H_4) + G_4 (1 + H_4)}{\Delta}$$

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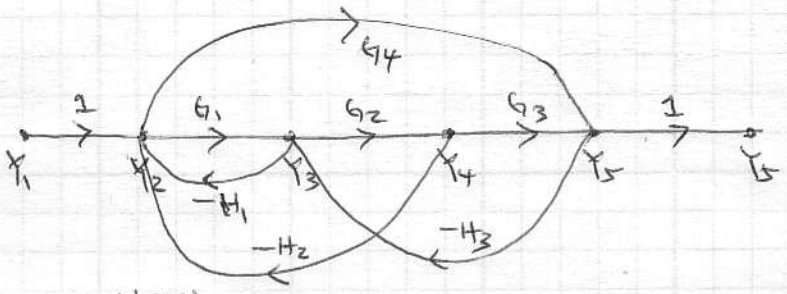
c)



Y_5 as output

Forward paths:

$M_1 = G_1 G_2 G_3$

$M_2 = G_4$

Independent loops:

$L_{11} = -G_1 H_1$

$L_{21} = -G_1 G_2 H_2$

$L_{31} = -G_2 G_3 H_3$

$L_{41} = G_4 H_1 H_3$

check for non touching loops $\Rightarrow$ None

$$\Delta = 1 + G_1 H_1 + G_1 G_2 H_2 + G_2 G_3 H_3 - G_4 H_1 H_3$$

$\Delta_1 = 1, \Delta_2 = 1$

$$\frac{Y_5}{Y_1} = \frac{G_1 G_2 G_3 + G_4}{\Delta}$$

Y_4 as output

Forward paths:

$M_1 = G_1 G_2$

All the loops - unchange $\Rightarrow$ Δ same

$\Delta_1 = 1$

$$\frac{Y_4}{Y_1} = \frac{G_1 G_2}{\Delta}$$
