ECEN 4413
Automatic Control Systems
Spring 2005

Midterm Exam #2

Choose any four out of five problems.
Please specify which four listed below to be graded:
1)_____; 2)_____; 3)_____; 4)_____;  

Name : ______________________________

Student ID: ________________________________

E-Mail Address:_______________________________________
**Problem 1:**
The equations that describe the dynamics of a motor control system are

\[
e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + K_b \frac{d\theta_m(t)}{dt}
\]

\[
T_m(t) = K_i i_a(t)
\]

\[
T_m(t) = J \frac{d^2 \theta_m(t)}{dt^2} + B \frac{d\theta_m(t)}{dt} + K\theta_m(t)
\]

\[
e_a(t) = K_a e(t)
\]

\[
e(t) = K_s \left[ \theta_r(t) - \theta_m(t) \right]
\]

a) Assign the state variables as \( x_1(t) = \theta_m(t) \), \( x_2(t) = \frac{d\theta_m(t)}{dt} \), and \( x_3(t) = i_a(t) \).

Express the state space representation in the form of

\[
\frac{dx(t)}{dt} = Ax(t) + B\theta_r(t), \quad \theta_m(t) = Cx(t).
\]

b) Find the transfer function \( G(s) = \Theta_m(s) / E(s) \) when the feedback path from \( \Theta_m(s) \) to \( E(s) \) is broken. Find the closed-loop transfer function, \( M(s) = \Theta_m(s) / \Theta_r(s) \).
**Problem 2:**

For the matrices

\[ A_1 = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \text{and} \quad A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \]

determine the functions of matrices \( e^{A_1} \) and \( A_2^{39} \).
**Problem 3:**
Derive a state space representation of the system given in the state diagram shown below in the form of \( \dot{x}(t) = Ax(t) + Br(t), \quad y(t) = Cx(t) \),

![State diagram image]
Problem 4:
Find an “equivalent” Jordan-canonical-form dynamical equation of
}\[
\begin{align*}
\dot{x}(t) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} u(t), \\
y(t) &= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} x(t).
\end{align*}
\]
**Problem 5:**
Show that two equivalent systems through similarity transformation are

a) zero-state equivalent (i.e., \( Ce^{A(t-\tau)} B + D \delta(t - \tau) = \overline{Ce}^{\overline{A}(t-\tau)} \overline{B} + \overline{D} \delta(t - \tau) \)), and

b) zero-input equivalent (i.e., \( Ce^{A(t-t_0)} x(t_0) = \overline{Ce}^{\overline{A}(t-t_0)} \overline{x}(t_0) \)).