Optimality Equations and New Rules for the Zone Control Chart

Eswar Sivaraman & Kenneth E. Case
School of Industrial Engineering & Management
Oklahoma State University, OK 74074
seswar@okstate.edu

Extended Abstract

The Zone Control Chart is a control chart devised for use in process control applications, and it offers advantages in ease of learning and simplicity of use with its relatively minor requirements for learning and implementation on the shop floor. Much like the standard X, or \( \overline{X} \) charts, the ZCC requires the user to segment the chart into three 1σ zones on either side of the center line (CL), assign zonal scores of \( \{0, \pm 1, \pm 2, \pm 4\} \) for either side of the CL, and cumulatively track scores for observed data. If the cumulative score exceeds \( \pm 4 \), an out-of-control (OOC) signal is effected, and should a point fall on the opposing side, the score is reset to the point value of the zone that the point falls in. To illustrate, suppose that the process is centered at zero, and the width of each zone is 1 unit, and that we observe the following points – \( \{0.6, 1.3, 1.45, -1.22, 0.33, -2.1, 2.44, 2.22\} \). The corresponding cumulative scores tracked on the ZCC are – \( \{0, 1, 2, -1, 0, -2, 2, 4\} \), and consequently, the ZCC will signal an OOC, much like how a standard Shewart chart would have reacted to this data set. However, while there are several superficial (dis)similarities with the standard Shewart charts, and although it has been demonstrated that the zone scores of \( \{0, \pm 1, \pm 2, \pm 4\} \) perform comparably in matching the ARL performance of the standard charts, there is a substantial gap in the theory driving the selection of these zone scores. It is disturbing to note that should the process produce 20 consecutive observations of, say, 0.6, the cumulative score on the ZCC would remain 0, and it would not signal an OOC, while a standard Shewart chart would have done so (run rule of seven points on one side of the CL). We address and solve this problem by presenting a optimization based approach for the selection of the zonal scores that would match, if not surpass the ARL performance of the standard Shewart chart. Also, the theory is generic in allowing the user to select and specify the desired zonal widths as opposed to working with just 1σ zone widths. We briefly outline our approach in the section below.

The standard Shewart chart with its runs rules (2/3, 4/5, etc.), was designed to ensure that the probability of a false alarm arising from say, the use of the 2 out of 3 rule, is roughly equal to the probability of a point falling outside the 3σ zone for an in-control process, namely, 0.00135. For ease of reference, we will label our zones as A, B, ..., H, where A = \(+\infty, +3\) , B = \(+3, +2\) , G = \(-2, -3\) , etc., and the zonal scores are \( \{a, b, c, d\} \), where \( d \) is the cumulative value that signals an OOC. Given this, how many points should fall (in any order) consecutively in zones D (0-1σ) and C (1-2σ) so that the probability of the occurrence of such a sequence roughly equals 0.00135? Note that the probability of a single point falling in zones C (zone score = a), and D (zone score = b) is 0.1359, and 0.3414, respectively. Suppose \( X_C \) and \( X_D \) are the number of points that fall consecutively (in some order) within zones C and D – the probability of such a sequence occurring is the multinomial probability \( P_i = \frac{(X_C + X_D)!}{(X_i!)(X_{-i}!)} (0.3414)^{X_C} (0.1359)^{X_D} \). If we expect the ZCC to react to this sequence as an OOC, then, \( P_i \leq 0.00135 \), solving for which we get \( X_C = 4 \) and \( X_D = 3 \) with \( P_i = 0.001193 \). Now, for the ZCC to react with an OOC signal, it follows that the cumulative score of \( (a*X_C + b*X_D) \geq d \), and this is the first optimality equation. We can thus proceed in considering similar point sequences for all other zone pairs to derive other optimality equations, which can be solved for either integer or real-valued zone scores. The disadvantage of using real-valued zone scores is the near impossibility of enumerating the state space of all possible cumulative scores to derive the ARL values. To this end, we will demonstrate the effectiveness of our approach and show results to the effect that zone scores of \( \{\pm 1, \pm 2, \pm 4, \pm 10\} \) offer superior ARL performance compared to the current practice of using \( \{0, \pm 1, \pm 2, \pm 4\} \) scores. Also, the extension of this approach to model user-specified flexible zone widths should encourage and further the use of this simple chart in industrial use.